'All mathematicians share... a sense of amazement over the infinite depth and mysterious beauty and usefulness of mathematics'

–**Martin Gardner**

MATHEMATICS
LESSON PLANS
GRADE 11 TERM 1

MESSAGE FROM NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE). We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

WHAT IS NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

WHAT ARE THE LEARNING PROGRAMMES?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

PROGRAMME ORIENTATION

Welcome!

The NECT FET Mathematics Learning Programme is designed to support teachers by providing:

- Lesson Plans
- Trackers
- Resource Packs
- Assessments and Memoranda
- Posters.

This Mathematics Learning Programme provides most of the planning required to teach FET Mathematics. However, it is important to remember that although the planning has been done for you, preparation is key to successful teaching. Set aside adequate time to properly prepare to teach each topic.

Also remember that the most important part of preparation is ensuring that you develop your own deep conceptual understanding of the topic. Do this by:

- working through the lesson plans for the topic
- watching the recommended video clips at the end of the topic
- completing all the worked examples in the lesson plans
- completing all activities and exercises in the textbook.

If, after this, a concept is still not clear to you, read through the section in the textbook or related teacher's guide, or ask a colleague for assistance. You may also wish to search for additional teaching videos and materials online. Some useful web links are listed at the end of each lesson plan.

Orientate yourself to this Learning Programme by looking at each component, and by taking note of the points that follow.

TERM 1 TEACHING PROGRAMME

1. In line with CAPS, the following teaching programme has been planned for FET Mathematics for Term 1:

Grade 10		Grade 11		Grade 12	
Topic	**No. of weeks**	**Topic**	**No. of weeks**	**Topic**	**No. of weeks**
Algebraic expressions	3	Exponents and Surds	3	Sequences and Series	3
Exponents	2	Equations and Inequalities	3	Functions (including inverses and logarithms)	4
Number Patterns	1	Number patterns	2	Euclidean Geometry	2
Equations and Inequalities	2	Analytical Geometry	3	Trigonometry	2
Trigonometry	3				
Total	11	Total	11	Total	11

* Note: CAPS amendments to be implemented in January 2019 require that in Grade 12, Euclidean Geometry be done in Term 1. The Grade 12 lesson plans reflect this. In order to ensure that you have the full set of topics for Grade 12, we have included the Topic of Finance at the back of the Grade 12 lesson plans. Finance is NOT done in Term 1.

2. Term 1 lesson plans and assessments are provided for eleven weeks for all three grades.
3. Each week includes 4,5 hours of teaching time, as per CAPS.
4. You may need to adjust the lesson breakdown to fit in with your school's timetable.

LESSON PLAN STRUCTURE

The Lesson Plan for each term is divided into topics. Each topic is presented in exactly the same way:

TOPIC OVERVIEW

1. Each topic begins with a brief **Topic Overview**. The topic overview locates the topic within the term, and gives a clear idea of the time that should be spent on the topic. It also

indicates the percentage value of this topic in the final examination, and gives an overview of the important skills and content that will be covered.

2. The **Lesson Breakdown Table** is essentially the teaching plan for the topic. This table lists the title of each lesson in the topic, as well as a suggested time allocation.
For example:

	Lesson title	Suggested time (hours)
1	Revision	2,5
2	Venn diagrams	2,5
3	Inclusive and mutually exclusive events; Complementary and Exhaustive events	1,5
4	Revision and Consolidation	1,5

3. The **Sequential Table** shows the prior knowledge required for this topic, the current knowledge and skills to be covered, and how this topic will be built on in future years.
   - Use this table to think about the topic conceptually:
     - Looking back, what conceptual understanding should learners have already mastered?
     - Looking forward, what further conceptual understanding must you develop in learners, in order for them to move on successfully?
   - If learners are not equipped with the knowledge and skills required for you to continue teaching, try to ensure that they have some understanding of the key concepts before moving on.
   - In some topics, a revision lesson has been provided.

4. The **NCS Diagnostic Reports**. This section is potentially very useful. It lists common problems and misconceptions that are evident in learners' NSC examination scripts. The Lesson Plans aim to address these problem areas, but it is also a good idea for you to keep these in mind as you teach a topic.

5. The **Assessment of the Topic** section outlines the formal assessment requirements as prescribed by CAPS for Term 1 (page 54).

Grade	Assessment requirements for Term 1 (as prescribed in CAPS)
10	Project/ Investigation; Test
11	Project/ Investigation; Test
12	Project/ investigation; Assignment; Test

The assessments are included in the Lesson plans and Resource Pack for each grade.

Mathematics School-based Assessment Exemplars – CAPS. Grade 12 Teacher Guide.

Some of the Grade 12 assessments come from: *Mathematics School-based Assessment Exemplars – CAPS. Grade 12 Teacher Guide. DBE, Pretoria*

A team of experts comprising teachers and subject advisors from different provinces was appointed by the DBE to develop and compile the assessment tasks in this document. The team was required to extract excellent pieces of learner tasks from their respective schools and districts. The panel of experts spent a period of four days at the DBE developing tasks based on guidelines and policies. Moderation and quality assurance of the tasks were undertaken by national and provincial examiners and moderators to ensure that they are in line with CAPS requirements.

Mathematics School-based Assessment Exemplars – CAPS. Grade 12 Teacher Guide. DBE, Pretoria, p4

You can access this document from various sites, including:
https://www.education.gov.za/SchoolBasedAssessmentTasks2014/tabid/611/Default.aspx

6. The glossary of **Mathematical Vocabulary** provides an explanation of each word or phrase relevant to the topic. In some cases, an explanatory sketch is also provided. It is a good idea to display these words and their definitions or sketches somewhere in the classroom for the duration of the topic. It is also a good idea to encourage learners to copy down this table in their free time, or alternately, to photocopy the Mathematical Vocabulary for learners at the start of the topic. You should explicitly teach the words and their meanings as and when you encounter these words in the topic.

INDIVIDUAL LESSONS

1.. Following the **Topic Overview**, you will find the **Individual Lessons**. Each lesson is structured in exactly the same way. The routine within the individual lessons helps to improve time on task, and therefore, curriculum coverage.

2. In addition to the lesson title and time allocation, each lesson plan includes the following:

   A. ***Policy and Outcomes***. This provides the CAPS reference, and an overview of the objectives that will be covered in the lesson.

   B. ***Classroom Management***. This provides guidance and support as you plan and prepare for the lesson.

   - Make sure that you are ready to begin your lesson, have all your resources ready (including resources from the Resource Pack), have notes written up on the chalk-board, and are fully prepared to begin.

- Classroom management also suggests that you plan which textbook activities and exercises will be done at which point in the lesson, and that you work through all exercises prior to the lesson.
- In some cases, classroom management will also require you to photocopy an item for learners prior to the lesson, or to ensure that you have manipulatives such as boxes and tins available.

The Learner Practice Table. This lists the relevant practice exercises that are available in each of the approved textbooks.

- It is important to note that the textbooks deal with topics in different ways, and therefore provide a range of learner activities and exercises. Because of this, you will need to plan when you will get learners to do the textbook activities and exercises.
- If you feel that the textbook used by your learners does not provide sufficient practice activities and exercises, you may need to consult other textbooks or references, including online references.
- The *Siyavula* Open Source Mathematics textbooks are offered to anyone wishing to learn mathematics and can be accessed on the following website: https://www.everythingmaths.co.za/read

***C. Conceptual Development**:*

This section provides support for the actual teaching stages of the lesson.

Introduction: This gives a brief overview of the lesson and how to approach it. Wherever possible, make links to prior knowledge and to everyday contexts.

Direct Instruction: Direct instruction forms the bulk of the lesson. This section describes the teaching steps that should be followed to ensure that learners develop conceptual understanding. It is important to note the following:

- Grey blocks talk directly to the teacher. These blocks include teaching tips or suggestions.
- Teaching is often done by working through an example on the chalkboard. These worked examples are always presented in a table. This table may include grey cells that are teaching notes. The teaching notes help the teacher to explain and demonstrate the working process to learners.
- As you work through the direct instruction section, and as you complete worked examples on the chalkboard, ensure that learners copy down:
    - formulae, reference notes and explanations
    - the worked examples, together with the learner's own annotations.
- These notes then become a reference for learners when completing examples on their own, or when preparing for examinations.
- At relevant points during the lesson, ensure that learners do some of the Learner Practice activities as outlined at the beginning of each lesson plan. Also, give

learners additional practice exercises and questions from past papers as homework. Ensure that learners are fully aware of your expectations in this respect.

D. ***Additional Activities / Reading****.* This section provides you with web links related to the topic. Get into the habit of visiting these links as part of your lesson preparation. As teacher, it is always a good idea to be more informed than your learners. If possible, organise for learners to view video clips that you find particularly useful.

TRACKER

1. A Tracker is provided for each grade. The Trackers are CAPS compliant in terms of content and time.
2. You can use the Tracker to document your progress. This helps you to monitor your pacing and curriculum coverage. If you fall behind, make a plan to catch up.
3. Fill in the Tracker on a daily or weekly basis.
4. At the end of each week, try to reflect on your teaching progress. This can be done with the HoD, with a subject head, with a colleague, or on your own. Make meaningful notes about what went well and what didn't. Use the reflection section to reflect on your teaching, the learners' learning and to note anything you would do differently next time.
5. These notes can become an important part of your preparation in the following year.

RESOURCE PACK, ASSESSMENT AND POSTERS

1. A Resource Pack with printable resources has been provided for each term.
2. These resources are referenced in the lesson plans.
3. Two posters have been provided as part of the FET Mathematics Learning Programme for Term 1.
4. Ensure that the posters are displayed in the classroom.
5. Try to ensure that the posters are durable and long-lasting by laminating them, or by covering them in contact adhesive.
6. Note that you will only be given these resources once. It is important for you to manage and store these resources properly. You can do this by:
   - Writing your school's name on all resources
   - Sticking resource pages onto cardboard or paper
   - Laminating all resources, or covering them in contact paper
   - Filing the resource papers in plastic sleeves once you have completed a topic.
7. Add other resources to your resource file as you go along.
8. Note that these resources remain the property of the school to which they were issued..

ASSESSMENT AND MEMORANDUM

In the Resource Pack you are provided with assessment exemplars and memoranda as per CAPS requirements for the term.

CONCLUSION

Teacher support and development is a complex process. For successful Mathematics teachers, certain aspects of this Learning Programme may strengthen your teaching approach. For emerging Mathematics teachers, we hope that this Learning Programme offers you meaningful support as you develop improved structure and routine in your classroom, develop deeper conceptual understanding in your learners and increase curriculum coverage.

Term 1, Topic 1: Topic Overview

EXPONENTS AND SURDS

A. TOPIC OVERVIEW

A

- This topic is the first of four topics in Term 1.
- This topic runs for three weeks (13,5 hours).
- It is presented over six lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 13,5 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Exponents and Surds is part of Algebraic expressions which counts 30% of the final Paper 1 examination.
- Exponents is part of Algebra which forms the foundation for all topics in Mathematics.
- A sound knowledge of exponents will assist learners in Calculus in Grade 12.

Breakdown of topic into 3 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Revision	1,5	5	Surds	2
2	Simplification using exponent laws	2	6	Surd equations	2
3	Rational exponents	2	7	Revision and Consolidation	2
4	Exponential equations	2			

B

SEQUENTIAL TABLE

GRADE 10 and Senior phase	GRADE 11	GRADE 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• 4 laws and two definitions of exponents • Simplify expressions using the laws of exponents • Solve equations using the laws of exponents • Establish between which two integers a surd lies	• Apply the laws of exponents to expressions involving rational exponents • Solve equations with rational exponents • Add, subtract, multiply and divide simple surds • Solve equations with surds	• Demonstrate an understanding of the definition of a logarithm • Solve real life problems involving exponents and logarithms.

C

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Exponents and Surds.

These include:

- Exponential laws need to be revised and practiced regularly throughout the FET phase
- Surd equations need to be covered thoroughly – learners must test whether their solutions satisfy the original equation.

It is important that you keep these issues in mind when teaching this section.

While teaching Exponents and Surds, always use the correct notation and mathematical language. Learners must be encouraged to do the same. A learner’s understanding of the concepts is more important than merely doing routine procedures.

ASSESSMENT OF THE TOPIC

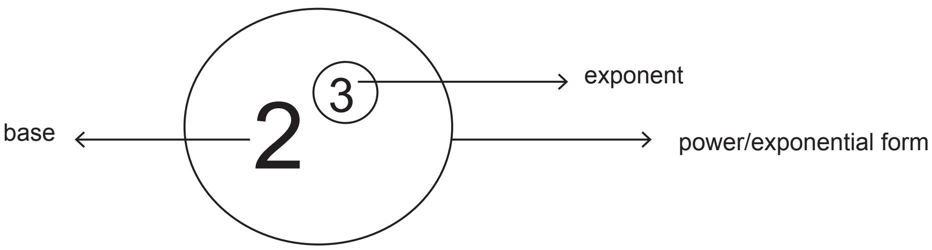
D

- CAPS formal assessment requirements for Term 1:
    - Investigation/Project
    - Test
- One test with memorandum and an investigation with a rubric are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). These are Resources 12 and 13 in the Resource Pack.
- The questions usually take the form of simplifying expressions and solving equations involving exponents and surds.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
exponential form	Constant or variable written as a power
exponent or index	The superscript digit that is written above the base and indicates the number of times the base is repeated in a multiplication calculation an exponent shows how many times a constant or variable is a factor
power	
prime factors	The factors that make up the number that are prime numbers numbers that have 1 two factors - 1and themselves as factors
surd	Number or quantity that cannot be expressed as the ratio of two integers An irrational number

TERM 1, TOPIC 1, LESSON 1

REVISION OF EXPONENT LAWS

Suggested lesson duration: 1,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	30

Lesson Objectives

By the end of the lesson, learners will have revised:

- the laws of exponents.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resources 1 and 2 ready for use during lesson.
4. Write the lesson heading on the board before learners arrive.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	3	1	5	Qu’s	9	1.2	5	1.2	8

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Many learners find this topic difficult. Ensure that quality time is spent on exponents to ensure confidence in learners.
2. Revise all related concepts covered in Grade 10.

DIRECT INSTRUCTION

1. Start the lesson by asking learners what rules they remember from exponents and to explain them with an example. Draw up a table as you go, repeating good explanations and topping up with any extra information you think may be necessary.
   A copy of this table is provided in the Resource Pack (Resource 1).
2. Ensure the summary has a minimum of the following in it and that learners write it in their books:

	Law	Example	Explanation
1	$x^a \times x^b = x^{a+b}$	$2^3 \times 2^2 \times 2$ $=2^{3+2+1}$ $=2^6$	When multiplying powers with like bases keep the bases the same and add the exponents.
2	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{6x^6}{2x^2} = 3x^4$	When dividing powers with like bases keep the base and subtract the exponents. Divide coefficients (numbers) as per normal.
3	$(x^a)^b = x^{ab}$	$(-2a^2b^3)^4$ $=(-2)^4 \times a^{2\times4} \times b^{3\times4})$ $=16a^8b^{12}$	When raising exponents to a power, keep the base and multiply the exponents.
4	$(xy)^a = x^a y^a$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$(a^4b)^3$ $= a^{12}b^3$ $\left(\frac{a^3}{b}\right)^3 = \frac{a^9}{b^3}$	When more than one base is raised to an exponent, each base is raised to the exponent. When a fraction is raised to an exponent, the numerator and denominator must be raised to that exponent.

3. Discuss the two definitions of exponents.
   A copy of this table is provided in the Resource Pack (Resource 2).

$x^0 = 1$	$(x^4 + 4)^0 + 3^0$ $= 1 + 1$ $= 2$	Any base raised to the power of zero is equal to 1. ($x \neq 0$ *as* 0^0 is undefined).
$x^{-a} = \dfrac{1}{x^a}$	$3x^{-2} = \dfrac{3}{x^2}$ and $\dfrac{3}{x^{-2}} = 3x^2$	A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.

4. Do two more types of examples that were covered in Grade 10. These examples include using prime factors and factorisation. There are two types of question:

Type 1: $\dfrac{6^{2x}.11^{2x}}{22^{2x-1}3^{2x}}$	
Point out that there is only one term in both the numerator and denominator. Ask: *Which bases can still be broken down into a product of prime factors?* (6 and 22). Ask: *What are these equal to as products of their prime factors?* (6 = 2 × 3 and 22 = 2 × 11).	
$\dfrac{6^{2x}.11^{2x}}{22^{2x-1}3^{2x}}$ $= \dfrac{(2.3)^{2x}11^{2x}}{(2.11)^{2x-1}3^{2x}}$	Composite numbers written as prime factors.
$= \dfrac{2^{2x}.3^{2x}.11^{2x}}{2^{2x-1}11^{2x-1}3^{2x}}$	Removing brackets by applying: ● the power raised to another power law ● more than one base raised to an exponent law.
$= 2^{2x-(2x-1)}\ 3^{2x-2x}\ 11^{2x-(2x-1)}$ $= 2^{2x-2x+1}\ 3^{2x-2x}\ 11^{2x-2x+1}$	Divide powers of the same base by subtracting exponents.

$= 2^1.3^0.11^1$	Simplify by collecting like terms.
$= 2 \times 11 = 22$	Simplify as far as possible using any definitions of exponents and calculating reasonable powers. Learners often ask when they need to calculate and when they can leave a number written in exponential form – the general rule is, if it can be worked out fairly quickly mentally it should be done. For example, 5^3, 2^4, 3^3 should all be calculated. 7^5 and 2^{10} would not be expected to be calculated.

5. Ask if there are any questions before covering the next type of question. Tell learners that this type will have more than one term in the numerator or denominator position.
   In this case, factorisation is always part of the process.

Type 2: $\dfrac{2.5^x + 5^{x-2}}{3.5^{x+1} - 7.5^{x-1}}$	
Point out that more than one term occurs within this fraction. In this case, both the numerator and denominator have two terms. The numerator and denominator both need to be factorised to create one term only so that simplification can take place.	
$\dfrac{2.5^x + 5^{x-2}}{3.5^{x+1} - 7.5^{x-1}}$	Tell learners it will make the factorising easier if each base that has more than one term in the exponent is re-written to show each term of the exponent with its base. Remind learners that it is like 'undoing' the simplification.
$= \dfrac{2.5^x + 5^x.5^{-2}}{3.5^x.5^1 - 7.5^x.5^{-1}}$	● Once this has been completed, ask: *How many terms are in the numerator?* (2). ● Ask: *Is there a common factor?* (Yes - $5x$). ● Repeat with the denominator. Point out that the common factor is the same in both the numerator and denominator – this is usually the case.
$= \dfrac{5^x(2 + 5^{-2})}{5^x(3.5 - 7.5^{-1})}$	Simplify.

$= \dfrac{2+5^{-2}}{3.5 - 7.5^{-1}}$	Tell learners they need to be very careful at this stage when dealing with negative exponents. As there are still two terms, learners cannot simply 'move the base to the other side'. It needs to be dealt with within the numerator or denominator.
$= \dfrac{2+\dfrac{1}{5^2}}{15 - 7.\left(\dfrac{1}{5}\right)}$	From this stage, learners need to be proficient in calculations with fractions.
$= \dfrac{2+\dfrac{1}{25}}{15-\dfrac{7}{5}}$ $= \dfrac{\dfrac{51}{25}}{\dfrac{68}{5}}$ $= \dfrac{51}{25} \div \dfrac{68}{5}$ $= \dfrac{51}{25} \times \dfrac{5}{68}$ $= \dfrac{51}{5} \times \dfrac{1}{68}$ $= \dfrac{51}{340}$	Tell learners that questions are not usually this complicated at the end but regular practice in working with fractions is excellent for their mathematics.

6. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

7. Give learners an exercise to complete on their own.

8. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=s1Gsu_0KcE4

https://www.youtube.com/watch?v=K_qN0RcMlQY

https://www.youtube.com/watch?v=QEPReLuHTQU&index=2&list=PLOaNAKtW5HLTSR0eRjIg-POD6HoSBoF07W&t=0s

TERM 1, TOPIC 1, LESSON 2

SIMPLIFICATION OF EXPRESSIONS USING THE LAWS OF EXPONENTS

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have: ● used the laws of exponents to simplify expressions.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the first few examples.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	5	2	6			1.3	8		

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Although Lesson 1 covered most of the skills required for this lesson, Grade 11 steps up a level and can be more complicated.
2. Give learners enough time to practice their skills in class where they can ask for assistance if necessary.
3. At the start of this lesson, work through all the questions that learners struggled with from the previous lesson's textbook exercise.

DIRECT INSTRUCTION

1. Tell learners you are going to work through more examples of simplifying expressions that involve the use of the laws. Say: *Although you have already done some examples like these, and practiced the skills in the revision exercise in the last lesson, it is important that you get as much practice as possible. Being confident in simplifying expressions with exponents is essential to many areas in mathematics.*
2. Learners should write the examples in their books and make notes as they do so.

Example and solution	Teaching notes:
1. $2^{3x-5}.\left(\frac{1}{4}\right)^{2x-3}.2^{x+2}$ $=2^{3x-5}.\left(\frac{1}{2^2}\right)^{2x-3}.2^{x+2}$ $=2^{3x-5}.(2^{-2})^{2x-3}.2^{x+2}$ $=2^{3x-5}.2^{-4x+6}.2^{x+2}$ $=2^3$ $=8$	Ask: *What needs to be done first? Remember that our aim is always to simplify so that we can use the laws of exponents.* (Change the 4 to a base 2 so all bases are the same). Once this has been done, say: *We need to use the negative exponent definition in reverse, so all the bases are also in standard position.* Ask: *What laws will be used to simplify further?* (Raising a power to another power and multiplying the exponents; multiplying powers of the same base and adding the exponents).

2. Find the value of 10^{x+3} if $10^x=1{,}5$ 10^{x+3} $=10^x.10^3$ $=1{,}5\times10^3$ $=1\,500$	Ask: *What can be done to 10^{x+3} so the other information can be substituted?* ('Undo' the simplification – rewrite the expression as the product of two equal bases).
3. Simplify without the use of a calculator: $125^{\frac{2}{3}}$ $125^{\frac{2}{3}}$ $=(5^3)^{\frac{2}{3}}$ $=5^2$ $=25$	Say: *'Without a calculator' usually means that it is possible to break down into more manageable numbers.* Ask: *What can be done to 125?* (It can be written as a power of 5). Once this has been completed, ask: *What law of exponents will be used now?* (Raising a power to another power and multiplying the exponents).
4. Simplify the following expressions: $\frac{2^{x+3}+2^x}{\sqrt{2^{2x}}}$ $=\frac{2^x.2^3+2^x}{\sqrt{2^x.2^x}}$ $=\frac{2^x(8+1)}{2^x}$ $=9$	Say: *The skills used in Example 2 are used again in this question.* *Because there are two terms, we need to factorise.* *To make the factorising easier, rewrite the simplified power where the exponents have been added into a product of the base.* Learners may find it more difficult to deal with the square root. Remind learners that any even exponent produces a perfect square. For example, $\sqrt{a^2}=a$ or $\sqrt{5^{2a}}=5^a$

5. $\frac{2^{4x+1}.9^{x}.6^{2x-1}}{12^{3x}.3^{x}}$ $=\frac{2^{4x+1}.(3^{2})^{x}.(2.3)^{2x-1}}{(2^{2}.3)^{3x}.3^{x}}$ $=\frac{2^{4x+1}.3^{2x}.2^{2x-1}.3^{2x-1}}{2^{6x}.3^{3x}.3^{x}}$ $=\frac{2^{6x}.3^{4x-1}}{2^{6x}.3^{4x}}$ $=2^{6x-6x}.3^{4x-1-4x}$ $=2^{0}.3^{-1}$ $=\frac{1}{3}$	Point out the difference between Example 5 and Example 4: The numerator and denominator are both made up of only one term. Ask: *What do we need to do first?* (Break all the composite numbers down into their prime factors). *Why?* (So that the laws of exponents can be used to simplify – like bases are required for some of the laws). Note: At each step, ask learners which laws or definitions will be used. This question covers all 4 laws and both definitions. Although many learners may notice that 2^{6x} could have been simplified without using the division of like bases, it is best to do it with them as done here to incorporate the zero-exponent definition.
6. $\frac{3.2^{m}-4.2^{m+2}}{2^{m}-2^{m+1}}$ $=\frac{3.2^{m}-4.2^{m}.2^{2}}{2^{m}-2^{m}.2^{1}}$ $=\frac{2^{m}(3-4.2^{2})}{2^{m}(1-2)}$ $=\frac{3-16}{1-2}$ $=\frac{-13}{-1}$ $=13$	Point out the difference between Example 6 and Example 5: The numerator and denominator both have two terms. Ask: *What does this information tell us?* (We need to factorise). Tell learners to do the first step on their own as that will assist them in the factorising process. Once this has been done, ask: *What is the common factor?* (2^{m})

3. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

4. Give learners an exercise to complete on their own. Consider finding a few extra questions from another textbook or a previous test to ensure that learners get as much practice as possible.

5. Walk around the classroom as learners do the exercise. Support learners where necessary.

TERM 1, TOPIC 1, LESSON 3

RATIONAL EXPONENTS

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have: • worked with rational exponents.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the first few examples.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3 4	6 7	6 7	10 10	1	11			1.3	11

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Rational exponents were new to learners in Grade 10. Rational exponents become a bigger part of this section in Grade 11.
2. Revise the types of questions covered in Grade 10 first to ensure that learners feel confident in those skills before starting Grade 11.

DIRECT INSTRUCTION

1. Ask: *What is a rational number?*
(Any number that can be written in the form $\frac{a}{b}$ where a,b ϵ Z but $b \neq 0$) .
2. Say: *If exponents are rational then they could be integers, but they could also be fractions. In today's lesson, we focus on rational exponents.*
3. Say: *We are going start by doing a few examples from Grade 10. When compound numbers are raised to a rational exponent, they can often be simplified by using prime factors and the rule for raising a power to another power. Remind learners you did this in Lesson 2.*

Example	Teaching notes	Solution
$8^{\frac{1}{3}}$	For each one of these questions, every composite number needs to be written in exponential form using prime factors.	$8^{\frac{1}{3}}$ $=(2^3)^{\frac{1}{3}}$ $=2$
$32^{\frac{3}{5}}$	Once that has been done, the 'raising a power to another power' law can be applied and, if necessary, any other exponential laws can be used to simplify further.	$32^{\frac{3}{5}}$ $=(2^5)^{\frac{3}{5}}$ $=2^3=8$
$\left(\frac{1}{81^{\frac{1}{4}}}\right)^3$		$\left(\frac{1}{81^{\frac{1}{4}}}\right)^3$ $=\frac{1}{(3^4)^{\frac{1}{4}}}^3$ $=\left(\frac{1}{3}\right)^3=\frac{1}{27}$

$20^{\frac{1}{2}}\times 10^{\frac{1}{2}}\times 2^{\frac{1}{2}}$		$20^{\frac{1}{2}}\times 10^{\frac{1}{2}}\times 2^{\frac{1}{2}}$ $=(2^{2}.5)^{\frac{1}{2}}\times (2.5)^{\frac{1}{2}}\times 2^{\frac{1}{2}}$ $=2.5^{\frac{1}{2}}\times 2^{\frac{1}{2}}.5^{\frac{1}{2}}\times 2^{\frac{1}{2}}$ $=2^{1+\frac{1}{2}+\frac{1}{2}}.5^{\frac{1}{2}+\frac{1}{2}}$ $=2^{2}.5=20$

4. Show what rational exponents really mean for the base.
   Ask learners to write the answers to the following questions.

$a^{\frac{1}{2}}\times a^{\frac{1}{2}}$	(a)	$x^{\frac{1}{3}}\times x^{\frac{1}{3}}\times x^{\frac{1}{3}}$	(x)

5. Ask learners to complete the following:

$\sqrt{a}\times\sqrt{a}$	$\left(=\sqrt{a^{2}}=a\right)$
$\sqrt[3]{x}\times\sqrt[3]{x}\times\sqrt[3]{x}$	$\left(=\sqrt[3]{x^{3}}=x\right)$

6. Ask: What conclusion can be made?

   $a^{\frac{1}{2}}=\sqrt{a}$ and $x^{\frac{1}{3}}=\sqrt[3]{x}$

7. Get learners to write the rule for changing a power with a rational exponent to a root:

$$base^{\frac{power}{root}}=\sqrt[root]{base^{power}} \text{ or } (\sqrt[root]{base})^{power}$$

$$a^{\frac{m}{n}}=\sqrt[n]{a^{m}} \text{ or } (\sqrt[n]{a})^{m}$$

8. Give learners a few examples to change from rational exponents to surds:

$3^{\frac{1}{2}}$	$13^{\frac{3}{4}}$	$x^{\frac{3}{5}}$	$x^{-\frac{3}{2}}$
$=\sqrt{3}(\sqrt[2]{3^{1}})$	$=\sqrt[4]{13^{3}}$	$=\sqrt[5]{x^{3}}$	$=\dfrac{1}{x^{\frac{3}{2}}}=\dfrac{1}{\sqrt{x^{3}}}$

9. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

10. Give learners an exercise to complete on their own.

11. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=w8jGGUYBqcA&index=3&list=PLOaNAKtW5HLTSR0eRjIgPOD-6HoSBoF07W&t=0s

TERM 1, TOPIC 1, LESSON 4

EXPONENTIAL EQUATIONS

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have: • solved exponential equations.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
7 8 9	12 13 16	3 4 5 8 9	7 8 8 12 13	2	13	1.4 1.5 1.6 1.7	9 11 13 14		

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners were introduced to exponential equations in Grade 9. Although the exponential equations have become increasingly difficult in Grade 10 and now Grade 11, the basics of solving exponential equations are the same.
2. As always, good knowledge from previous years makes a positive difference for learners.

DIRECT INSTRUCTION

1. Write $3^{x+2} = 3^{4-x}$. Ask: *What makes this an easy exponential equation to solve?* (The bases are the same, therefore the exponents are equal).
2. Tell learners to solve the equation.

$$3^{x+2} = 3^{4-x}$$

$$\therefore x + 2 = 4 - x$$

$$2x = 2$$

$$x = 1$$

3. Say: *You should always check answers to equations. You can use your calculator to check; this has the added benefit of giving you practice with your calculator. Check the answer now by substituting into the left-hand side and right-hand side to confirm that they are equal.*

LHS	RHS
$= 3^{x+2}$ $= 3^{1+2}$ $= 3^3$ $= 27$	$= 3^{4-x}$ $= 3^{4-1}$ $= 3^3$ $= 27$
LHS = RHS $\quad \therefore x = 1$ is correct	

4. Do some examples that learners should be able to do using knowledge from Grade 10. Learners should write the examples in their books and make notes as they do so.

Examples Solve the following:	Teaching notes:
1. $2^x = 0{,}25$ $2^x = \frac{1}{4}$ $2^x = \frac{1}{2^2}$ $2^x = 2^{-2}$ $\therefore\ x = -2$	Remind learners that we need the bases to be equal when solving exponential equations. Ask: *How can we do this?* (Change the decimal into a common fraction). From this point, learners should be able to tell you the steps that follow.
2. $\frac{1}{3}.(3)^{x+1} = \frac{1}{9}$ $3.(3)^{x+1} = 1$ $3^{1+x+1} = 1$ $3^{2+x} = 1$ $3^{2+x} = 3^0$ $\therefore\ 2 + x = 0$ $\therefore\ x = -2$	Ask: *What can I do to simplify this equation?* (Multiply both sides by the lowest common denominator – 9). Ask: *What can be done to simplify further?* (Simplify the left-hand side of the equation using the rule for multiplying powers of the same base by adding the exponents). Remind learners that we require equal bases. Ask: *How can we achieve this?* (Use the power zero to change 1 into a base 3).
3. $12\left(\frac{1}{4}\right)^x = 12\left(\frac{1}{4}\right)^2$ $\left(\frac{1}{4}\right)^x = \left(\frac{1}{4}\right)^2$ $\therefore\ x = 2$	This is simpler than it looks. The reason for using it as an example is to discuss why learners cannot simplify the denominator of 4 with the 12. Point out that the '4' is part of a power that has an exponent which means there won't only be one of them (unless the exponent is 1, which it is not). Ask: *What can you do to simplify this equation?* (Divide both sides by 12).

5. Ask learners if there are any questions before moving on.

6. Say: *Now we will look at equations with rational exponents where the base is the unknown rather than the exponent.*

7. Write the following examples on the board and do them in full with learners. These are examples from Grade 10.

Examples Solve the following:	Teaching notes:
1. $x^{\frac{1}{2}} = 7$ $\left(x^{\frac{1}{2}}\right)^2 = 7^2$ $x = 49$	When solving for a base that has a rational exponent, raise both sides to the power of the denominator. Use the ‘raising a power to another power rule’ to simplify and get the base on its own. This can only be done if the base with the rational exponent is by itself. In example 3, both sides will be divided by 3 first to ensure this is the case.
2. $x^{\frac{1}{5}} = 2$ $\left(x^{\frac{1}{5}}\right)^5 = 2^5$ $x = 32$	
3. $3x^{\frac{1}{3}} = 12$ $\frac{3x^{\frac{1}{3}}}{3} = \frac{12}{3}$ $x^{\frac{1}{3}} = 4$ $\left(x^{\frac{1}{3}}\right)^3 = 4^3$ $x = 64$	

8. Leave these answers on the board and discuss:

   If an unknown base is raised to a rational exponent in the form $\frac{1}{a}$, where a is an even number, it is not possible for the base to be a negative number.

9. Use the following examples to show why this is the case:

$x^{\frac{1}{2}} = 5$ $\left(x^{\frac{1}{2}}\right)^2 = 5^2$ $\therefore\ x = 25$	$x^{\frac{1}{4}} = 2$ $\left(x^{\frac{1}{4}}\right)^4 = 2^4$ $\therefore\ x = 16$
Use substitution to check the answers.	
$x^{\frac{1}{2}} = 25^{\frac{1}{2}}$ $= \sqrt{25}$ $= 5 = \text{RHS}$	$x^{\frac{1}{4}} = 16^{\frac{1}{4}}$ $= \sqrt[4]{16}$ $= 2 = \text{RHS}$
Point out that if the base was negative, there would be a root of a negative number and this is not possible when the root is even. It is possible when the root is an odd. For example, $\sqrt[3]{-8} = -2$.	

10. Ask if there are any questions before moving on to more complicated examples.

11. Point out that the three examples just completed all had a rational exponent where 1 was in the numerator position.
    Say: *Now we need to look at equations where the rational exponent of the unknown base has a numerator other than 1.*

12. Ask: *What can you tell me about an equation with an exponent of 2 in it, such as* $x^2 = 16$? (There will be two solutions).
    Tell learners to note that 2 is an even number.

13. Before doing an example, tell learners the following rule:
    If an equation is in the form: $x^{\frac{a}{b}} = y$:
    - there will be a positive and negative solution if a is even and b odd
    - there will be one solution if a is odd.

    Tell learners to write this rule in their books. Point out that if b was even, the rational exponent could be simplified.

14. Write the following example on the board and do it in full with learners.

$x^{\frac{2}{3}} - 16 = 0$	Teaching notes:
$x^{\frac{2}{3}} - 16 = 0$ $x^{\frac{2}{3}} = 16$ $x^{\frac{2}{3}} = 2^4$ $\left(x^{\frac{2}{3}}\right)^3 = (2^4)^3$ $x^2 = 2^{12}$ $\sqrt{x^2} = \pm\sqrt{2^{12}}$ $x = \pm 2^6$ $x = \pm 64$	Ask: *What needs to be done first to solve for* x*?* (Add 16 to both sides). Say: *As 16 is a composite number that can be written as a power, this should be done to make the question simpler for the next steps.* Point out that a is even b is odd $\therefore$ there will be two possible solutions (±). Ask: *What is the next step toward getting x on its own?* (Raise both sides to the power of 3). Note: although both sides could be raised to the power of $\frac{3}{2}$, rather do it in smaller steps at this stage. This also makes it clearer why there should be two possible solutions. Ask: *What is the next step toward getting* x *on its own?* (Find the square root of both sides). Remind learners: If the rational exponent has an even numerator: ± ans

15. Ask learners if they have any questions before moving on to equations that require factorising.

16. Work through two examples with learners. Learners should write the examples in their books and make notes as they do so.

These two examples require finding the highest common factor. There are exponential equations of higher order that will only be done in the next topic, Equations and Inequalities as it requires using the k-method (substitution method) and factorising a trinomial.

Example:	Teaching notes:
1. $3^{x+2} + 3^{x+3} - 3^{x} = 105$ $3^{x}.3^{2} + 3^{x}.3^{3} - 3^{x} = 105$ $3^{x}(3^{2} + 3^{3} - 1) = 105$ $3^{x}(35) = 105$ $\frac{3^{x}(35)}{35} = \frac{105}{35}$ $3^{x} = 3$ $\therefore\ x = 1$	Ask: *How many terms are there on the left-hand side?* (3). Say: *We need to factorise. We need to use the approach that was used in simplifying expressions with more than one term. How did we do this?* (Use the inverse of law 1 to write each base with a single exponent). Once this has been done, ask: *What do we need to do in the next step?* (Factorise by taking out a common factor).
2. $2^{x-2} = 5 - 2^{x}$ $2^{x-2} + 2^{x} = 5$ $2^{x}.2^{-2} + 2^{x} = 5$ $2^{x}(2^{-2} + 1) = 5$ $2^{x}\left(\frac{1}{4} + 1\right) = 5$ $2^{x}\left(\frac{5}{4}\right) = 5$ $2^{x}(5) = 20$ $2^{x} = 4$ $2^{x} = 2^{2}$ $\therefore\ x = 2$	Point out that there is a power with the variable that needs to be solved on each side of the equation and that we will need to have all the terms with a variable in it on one side. Once this has been completed the process will be the same as in Example 1.

17. Remind learners they should always check their solution to confirm that it will make the equation true.

18. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

19. Give learners an exercise to complete on their own.

20. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=IljeUG50DTU

https://www.youtube.com/watch?v=-OFC9iRyO1o

https://www.chilimath.com/lessons/advanced-algebra/solving-exponential-equations-without-logarithms/

https://www.youtube.com/watch?v=evPu1YfQiKw

TERM 1, TOPIC 1, LESSON 5

SIMPLIFICATION OF SURDS

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have: ● simplified surds.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5 6	9 11	10 11 12 13 14	16 16 17 17 17	3 4	15 16	1.8 1.9 1.10	16 18 20	1.4 1.5	16 18

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Surds are directly linked to exponents. We looked at the relationship briefly in the lesson on rational exponents.
2. Surds appear more and more frequently in solutions in Grade 11 and Grade 12. Learners need to know how to work with them.

DIRECT INSTRUCTION

1. Start the lesson by writing $\sqrt{49}$ and $\sqrt{10}$ on the chalkboard.
2. Ask: *Can these be simplified?*
   ($\sqrt{49}$ can – it is equal to 7; $\sqrt{10}$ cannot be simplified)
3. Tell learners that $\sqrt{10}$ is a surd as it represents an irrational number. Writing in the form $\sqrt{10}$ makes it more accurate. If we used our calculators to find $\sqrt{10}$, it would be essential to round the answer and therefore it would not be entirely accurate.
4. Leaving a number in its root form to express the exact value is a surd. Tell learners to write this in their books.
5. There are certain laws we need to apply when working with surds. Go through these with learners now. Tell them to write the laws in their books, making notes as they do so.

	Law (rule) and example:	Explanation/teaching notes
1	$\sqrt[n]{xy} = \sqrt[n]{x} \times \sqrt[n]{y}$ $\sqrt{18} = \sqrt{9} \times \sqrt{2}$ $= 3\sqrt{2}$	When surds are multiplied, they can be split apart and rooted individually. This is useful to simplify surds. $\sqrt{18}$ is not in its simplest form because a perfect square is a factor of 18. The key idea is to check if a composite number can be written as a product of a square number and another factor.

2	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\frac{\sqrt{12}}{\sqrt{121}}$ $= \frac{\sqrt{12}}{\sqrt{121}}$ $= \frac{\sqrt{12}}{11}$ $= \frac{\sqrt{4}.\sqrt{3}}{11}$ $= \frac{2\sqrt{3}}{11}$	When surds are divided, they can be split apart and rooted individually. This is useful to simplify surds. 121 is a perfect square, so when the split has been made that can be simplified. Note that $\sqrt{12}$ is not yet in its simplest form. The first law can be used to simplify it further.
3.	$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$ $5\sqrt{6} - 2\sqrt{6}$ $= 3\sqrt{6}$	If the root is of the same number or variable surds are treated the same as like terms. They can be added or subtracted, and the root remains the same.
4	$\sqrt[mn]{x} = \sqrt[m]{\sqrt[n]{x}}$ $\sqrt[6]{64}$ $= \sqrt[3\times2]{64}$ $= \sqrt[3]{\sqrt{64}}$ $= \sqrt[3]{8} = 2$	When taking the root of a root, it is the same as taking the single root to the product of both roots. This is to make a calculation simpler if no calculator is allowed. Instead of taking the 6th root, the solution can be found by finding the cube root of the square root (because 3 × 2 = 6)
	It is accepted practice that an answer is not completely simplified if there is a root/ surd in the denominator. This would need to be taken a step further. This is called rationalising the denominator.	
	$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$ $\frac{5}{\sqrt{11}}$ $= \frac{5}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$ $= \frac{5\sqrt{11}}{11}$	To rationalise the denominator, the aim is to multiply by a version of 1 that will remove the surd from the denominator. Remind learners that $\sqrt{a} \times \sqrt{a} = a$. It is this concept that is being used. In the example, $\sqrt{11}$ needs to be removed from the denominator. To do this, multiply by $\frac{\sqrt{11}}{\sqrt{11}}$, which equals one and therefore will not change the answer.

<table>
<tr><td colspan="2">Before doing the second type of question that requires rationalising of the denominator, first do the following two examples on the board:
Ask learners to multiply out and simplify the following:
$(\sqrt{5} - 2)(\sqrt{5} + 2)$
$= 5 + 2\sqrt{5} - 2\sqrt{5} - 4$
$= 1$
$(\sqrt{7} - 8)(\sqrt{7} + 8)$
$= 7 + 8\sqrt{7} - 8\sqrt{7} - 64$
$= -57$
Say: Note that by multiplying a two-term expression involving a surd by the same expression but with the opposite sign to create a difference of two squares, the surd can be eliminated. (The correct term is the conjugate which you can share with learners if you like. $x + y$ is the conjugate of $x - y$).</td></tr>
<tr><td colspan="2">$\frac{a}{a + b\sqrt{n}}$ → multiply by $\frac{a - b\sqrt{n}}{a - b\sqrt{n}}$ to rationalise the denominator
$\frac{a}{a - b\sqrt{n}}$ → multiply by $\frac{a + b\sqrt{n}}{a + b\sqrt{n}}$ to rationalise the denominator</td></tr>
<tr><td>Examples
1.
$\frac{3}{2 + \sqrt{2}}$
$= \frac{3}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$
$= \frac{3(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}$
$= \frac{6 - 3\sqrt{2}}{4 - 2}$
$= \frac{6 - 3\sqrt{2}}{2}$</td><td>For both examples:
Ask: what must we multiply by to rationalise the denominator?
$\frac{2 - \sqrt{2}}{2 - \sqrt{2}}$ for example 1; $\frac{5 + \sqrt{7}}{5 + \sqrt{7}}$ for example 2.
Work through the other steps slowly, ensuring learners understand as you do so.</td></tr>
</table>

2. $\frac{-4}{5-\sqrt{7}}$ $= \frac{-4}{5-\sqrt{7}} \times \frac{5+\sqrt{7}}{5+\sqrt{7}}$ $= \frac{-4(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})}$ $= \frac{-20-4\sqrt{7}}{5-7}$ $= \frac{-20-4\sqrt{7}}{-2}$ $= \frac{-20}{-2} + \frac{4\sqrt{7}}{2}$ $= 10 + 2\sqrt{7}$	This example required further simplifying because the number in the denominator is a factor of both terms in the numerator. Remind learners of their Grade 8 algebra – when dividing more than one term by one term, the term in the denominator 'belongs' to all terms in the numerator.

6. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
7. If you think learners need a few more examples before completing an exercise on their own, do the examples that are in the textbook you use. If you think learners are ready, ask them to study the examples in their books before they complete the exercise and ask you questions if any of them are unclear.
8. Give learners an exercise to complete on their own.
9. Walk around the classroom as learners do the exercise. Support learners where necessary

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=9EJ2UYtg57k

https://www.youtube.com/watch?v=iTt3TVjsjuY

TERM 1, TOPIC 1, LESSON 6

SURD EQUATIONS

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have: • solved surd equations.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
10	18	15	18	5 3	17 26	1.11 1.12	21 22	1.6	22

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Surd equations are new in Grade 11 and will be assessed again in Grade 12.

DIRECT INSTRUCTION

1. Say: *Now that you have had practice with simplifying surds and the expressions, we are going to look at solving equations with surds in them.*

The examples below start with basic equations that produce valid solutions then build up to equations where a solution may be invalid due to the process of squaring both sides. This is explained in the examples.

2. Work through examples with learners.
   Learners should write the examples in their books and make notes as they do so.

Example:	Teaching notes:
Say: *The key to solving all equations with surds is to isolate the surd then square both sides to remove the root sign.* Say: *Always check the solution. This will be explained as we progress through the examples.*	
1. $\sqrt{x} - 3 = 5$ $\sqrt{x} = 5 + 3$ $\sqrt{x} = 8$ $(\sqrt{x})^2 = 8^2$ $x = 64$ Check: LHS: $\sqrt{64} - 3$ $= 8 - 3$ $= 5 =$ RHS	Ask: *what needs to be done to isolate the root sign?* (add 3 to both sides). Say: *now both sides can be squared – the inverse operation to finding the square root.* Say: *Finally, we use substitution to check the solution.*

2. $\sqrt{x+8} = 3$ $(\sqrt{x+8})^2 = 3^2$ $x + 8 = 9$ $x = 1$ Check: LHS: $\sqrt{1+8}$ $= \sqrt{9}$ $= 3 =$ RHS	Ask: *What is the first step to solve this equation?* (The root sign is already isolated, so square both sides). Once the solution has been found, say: *Finally, we use substitution to check the solution.*
3. $1 + \sqrt{2x+3} = 6$ $\sqrt{2x+3} = 5$ $(\sqrt{2x+3})^2 = 5^2$ $2x + 3 = 25$ $2x = 22$ $x = 11$ Check: $1 + \sqrt{2(11)+3}$ $= 1 + \sqrt{25}$ $= 1 + 5$ $= 6 =$ RHS	Ask: *What needs to be done to isolate the root sign?* (Subtract 1 from both sides). Say: *Now both sides can be squared – the inverse operation to square rooting.* Once the solution has been found, say: *Finally, we use substitution to check the solution.*
4. $\sqrt{a-5} = -2$ $(\sqrt{a-5})^2 = (-2)^2$ $a - 5 = 4$ $a = 1$ Check: LHS $= \sqrt{1-5}$ $= \sqrt{-4}$ (no solution) $\therefore a \neq 1$	Ask: *What is the first step to solve this equation?* (The root sign is already isolated, so square both sides). Once the solution has been found, say: *Finally, we use substitution to check the solution.* Say: *The square root of a negative number is not possible. Therefore, a cannot be equal to -1.* Ask: *If you look at the original equation that needed to be solved, can you see why there isn't a possible solution?* (The principal square root of a number can only be positive). Say: *This shows that there was never going to be a possible solution.*

This may require some extra explanation which will assist learners in understanding why some solutions may not work and why we always need to check solutions after squaring both sides.

The reason lies in the process of the squaring itself. When you raise a number to an even power, a false solution could be introduced because the result of an even power is always a positive number.

3^2 and $(-3)^2$ both equal 9, and 2^4 and $(-2)^4$ both equal 16.

In the above example, when −2 was squared the answer was 4; this 'artificially' turned the quantity positive. Therefore, it seemed possible to solve the equation and find what seemed like a reasonable solution for a.

The problem was solved as if the equation was $\sqrt{a-5} = 2$

1. $x + 4 = \sqrt{x+10}$ $(x+4)^2 = (\sqrt{x+10})^2$ $x^2 + 8x + 16 = x + 10$ $x^2 + 7x + 6 = 0$ $(x+6)(x+1) = 0$ $x = -6$ or $x = -1$ Check: $(x = -6)$ LHS = $-6 + 4 = -2$ RHS = $\sqrt{-6+10} = \sqrt{4} = 2$ LHS ≠ RHS ∴ x = –6 is NOT a solution Check: $(x = -1)$ LHS = $-1 + 4 = 3$ RHS = $\sqrt{-1+10} = \sqrt{9} = 3$ LHS = RHS $\therefore x = -1$	Ask: *What is the first step to solve this equation?* (The root sign is already isolated, so square both sides). Remind learners that when squaring two terms, they should use FOIL and NOT square each term. $(x+4)^2 \neq x^2 + 16$ Point out that once both sides have been squared a quadratic equation has appeared. Remind learners of the basic steps to solve a quadratic equation: All terms need to be on one side of the equation and equal to zero; factorise; find both possible solutions. Point out that in this equation, it isn't easy to see that there may be an impossible solution because there are variables on both sides. Once the solutions have been found, say: *Finally, we use substitution to check the solution.* *Note that only one of the solutions is valid.*

2. $4+\sqrt{x+2} = x$ $\sqrt{x+2} = x-4$ $(\sqrt{x+2})^2 = (x-4)^2$ $x + 2 = x^2 - 8x + 16$ $0 = x^2 - 9x + 14$ $0 = (x-7)(x-2)$ $x = 7$ or $x = 2$ Check: LHS = $4+\sqrt{7+2}$ $= 4+\sqrt{9} = 7$ RHS = 7 LHS = RHS $\therefore\ x = 7$ LHS = $4+\sqrt{2+2}$ $= 4+\sqrt{4} = 6$ RHS = 2 LHS ≠ RHS $\therefore\ x = 2$ is NOT a solution $\therefore\ x = 7$	Ask: *What needs to be done to isolate the root sign?* (Subtract 4 from both sides). Say: *Now both sides can be squared.* Once the solutions have been found, say: *Finally, we use substitution to check the solution.* *Note that only one of the solutions is valid.*

3. Ask learners if they have any questions. Summarise the steps involved in solving equations with surds. Learners should write it in their books.
   - Isolate the surd
   - Square both sides
   - Solve for x
   - Check your answer(s).

4. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

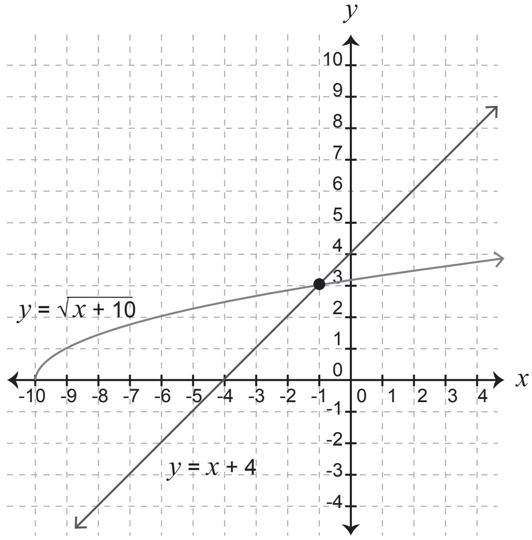
5. Give learners an exercise to complete on their own.

6. Walk around the classroom as learners do the exercise. Support learners where necessary.

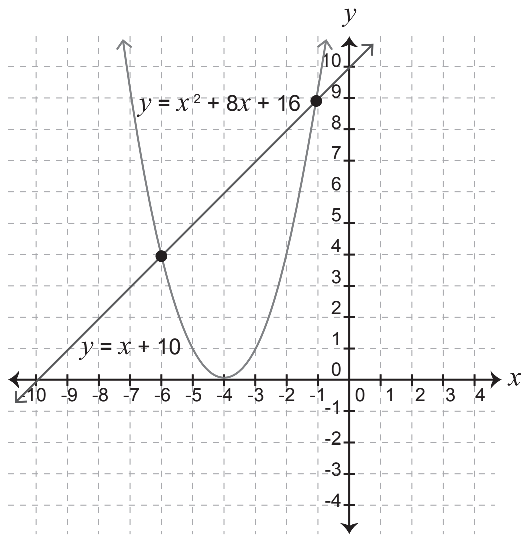
7. Once the exercise has been completed and marked, share the following information with learners (time permitting and if you feel learners need further explanation of why one or more answers could be invalid):

Although learners have not learned about inverse functions yet, you can show them the following to get a better idea using a visual representation of what is being solved.
This includes an understanding that when ANY equation is being solved algebraically the solution represents where the function represented on the left-hand side intersects the function represented on the right-hand side.

Many learners are not told this in earlier grades and it often comes as a surprise to them.
Using the example from earlier,
$x + 4 = \sqrt{x + 10}$, where the solutions found were $x = -6$ or $x = -1$.
The functions on each side are represented below:



Once each side was squared, the equation looked like this:
$x^2 + 8x + 16 = x + 10$.
The functions on each side are represented below:



Point out that the solutions found are the points of intersection for the SECOND step rather than the points of intersection for the ORIGINAL functions.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=Wvrfv5pZj8Y

https://www.youtube.com/watch?v=cHqI9rqhbMU

TERM 1, TOPIC 1, LESSON 7

REVISION AND CONSOLIDATION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	30

Lesson Objectives

By the end of the lesson, learners will have revised:

- the laws of exponents.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the first few examples.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	18 19	Rev	20	Qu’s	18	Rev	22	1.8	26

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Ask learners to recap what they have learned in this section. Spend time pointing out issues that you know are important as well as problems that you encountered from your own learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

This lesson is made up of fully worked examples covering most of the concepts in this topic. The examples are from a past examination. As you work through these with the learners, it is important to frequently talk about as many concepts as possible.
For example, use the words exponent, surd, prime factors, factorise and the term rationalise the denominator.

Say: *I am going to do an entire Exponents question from the 2016 final examination and the Eastern Cape 2015 final examination with you. You should write them down as I do them, taking notes at the same time.*

Questions:

1. Simplify fully, WITHOUT using a calculator:

   a) $\dfrac{5^{a-2}.2^{a+2}}{10^{a}-10^{a-1}.2}$

   b) $\dfrac{\sqrt{27m^{6}}-\sqrt{48m^{6}}}{\sqrt{12m^{6}}}$

2. WITHOUT using a calculator, show that: $\dfrac{2}{1+\sqrt{2}}-\dfrac{8}{\sqrt{8}}=-2$

Teaching notes:

1a)

Remind learners that as there are two terms in the denominator they should factorise. This usually implies that it is best to write each base to the power of only one exponent to assist in simplifying. Point out the importance of being confident in working with fractions.

1b)

Tell learners that each of the numbers in the root signs needs to be broken into a product of two factors where one is a perfect square. The square root of these should then be found. Discuss the process of finding the square root of exponents. Once each term is in its simplest form, learners should be able to tell you that they can take out a common factor.

2.

This question requires rationalising the denominators. Learners may need a quick reminder of how to do this – particularly for the first term which is more difficult. Once the $\sqrt{8}$ is on its own, remind learners to simplify it by writing 8 as a product of two factors where one of them is a perfect square.

Solutions:

1a) $\dfrac{5^{a-2}.2^{a+2}}{10^{a} - 10^{a-1}.2}$

$= \dfrac{5^{a}.5^{-2}.2^{a}.2^{2}}{10^{a} - 10^{a}.10^{-1}.2}$

$= \dfrac{(5.2)^{a}.5^{-2}.2^{2}}{10^{a}(1 - 10^{-1}.2)}$

$= \dfrac{10^{a}.5^{-2}.2^{2}}{10^{a}(1 - 10^{-1}.2)}$

$= \dfrac{5^{-2}.2^{2}}{1 - 10^{-1}.2}$

$= \dfrac{\frac{1}{5^{2}}.2^{2}}{1 - \frac{1}{10}.2}$

$= \dfrac{\frac{4}{25}}{1 - \frac{1}{5}}$

$= \dfrac{\frac{4}{25}}{\frac{4}{5}}$

$= \dfrac{4}{25} \div \dfrac{4}{5}$

$= \dfrac{4}{25} \times \dfrac{5}{4}$

$= \dfrac{1}{5}$

1b) $\dfrac{\sqrt{27m^6}-\sqrt{48m^6}}{\sqrt{12m^6}}$

$=\dfrac{\sqrt{9.3m^6}-\sqrt{16.3m^6}}{\sqrt{4.3m^6}}$

$=\dfrac{3.\sqrt{3}m^3-4.\sqrt{3}m^3}{2\sqrt{3}m^3}$

$=\dfrac{\sqrt{3}m^3(3-4)}{2.\sqrt{3}m^3}$

$=\dfrac{-1}{2}$

2. LHS $=\dfrac{2}{1+\sqrt{2}}-\dfrac{8}{\sqrt{8}}$

$=\dfrac{2}{1+\sqrt{2}}\times\dfrac{1-\sqrt{2}}{1-\sqrt{2}}-\dfrac{8}{\sqrt{8}}\times\dfrac{\sqrt{8}}{\sqrt{8}}$

$=\dfrac{2-2\sqrt{2}}{1-2}-\sqrt{8}$

$=\dfrac{2-2\sqrt{2}}{1-2}-2\sqrt{2}$

$=2+2\sqrt{2}-2\sqrt{2}$

$=-2=$ RHS

Questions:

1. Simplify the following expressions without the use of a calculator:
   a) $\dfrac{\sqrt{50}+\sqrt{8}}{7\sqrt{2}}$
   b) $\left[\dfrac{16x^{\frac{-5}{6}}}{81\sqrt{x}}\right]^{\frac{-3}{4}}$
2. Solve for x: $27^{x^2+x}=3^{3x^2}\times 9$
3. If $5^{-x}=10$, determine the value of $\dfrac{2^{x-1}+2^{x+1}}{5\times 10^{x}}$ without the use of a calculator.

Teaching notes:

1a)

As an example like this was done in the previous question, ask learners to talk you through the steps.

1b)

Say: *You should first simplify within the bracket before using the rule to raise a power to another power.*

Ask: *How will 16 and 81 be written as a product of their prime factors?*

Ask: *How will $\sqrt{x}$ be written as a power?*

Once this has been done, the exponent outside can be distributed to each base using multiplication.

Finally, the basic rules of exponents can be used to simplify fully.

2.

Ask: *What is the main rule needed to solve exponential equations?*

(Bases must be equal).

Ask learners to break down any composite numbers into their prime factors. Once bases are the same on each side, exponents are equal.

Solve the equation as usual. For this equation, it seems at first that the equation will be quadratic but, as learners will see, the terms with the x^2 are no longer there once like terms have been collected.

3.

Suggest that the focus should first be on simplifying the expression given. Once this has been completed, they should expect to find 5^{-x} in their expression which should then be replaced by 10. A similar question was covered in 1.a) of the previous example. Remind learners to expect to factorise because of the two terms in the numerator and to write each base to the power of one exponent only to assist them in this process.

Solutions:

1a) $\dfrac{\sqrt{50}+\sqrt{8}}{7\sqrt{2}}$

$= \dfrac{\sqrt{25.2}+\sqrt{4.2}}{7\sqrt{2}}$

$= \dfrac{5\sqrt{2}+2\sqrt{2}}{7\sqrt{2}}$

$= \dfrac{7\sqrt{2}}{7\sqrt{2}}$

$= 1$

1b) $\left[\dfrac{16x^{\frac{-5}{6}}}{81\sqrt{x}}\right]^{\frac{-3}{4}}$

$= \left[\dfrac{2^4x^{\frac{-5}{6}}}{81\sqrt{x}}\right]^{\frac{-3}{4}}$

$= \dfrac{2^{-3}.x^{\frac{5}{8}}}{3^{-3}.x^{\frac{-3}{8}}}$

$= \dfrac{3^3.x^{\frac{3}{8}}.x^{\frac{3}{8}}}{2^3}$

$= \dfrac{27x}{8}$

2.
$$27^{x^2+x} = 3^{3x^2} \times 9$$
$$(3^3)^{x^2+x} = 3^{3x^2} \times 3^2$$
$$3^{3x^2+3x} = 3^{x^2+2}$$
$$\therefore\ 3x^2 + 3x = 3x^2 + 2$$
$$3x = 2$$
$$x = \frac{2}{3}$$

3.
$$\frac{2^{x-1} + 2^{x+1}}{5 \times 10^x}$$
$$= \frac{2^x.2^{-1} + 2^x 2^1}{5 \times (2.5)^x}$$
$$= \frac{2^x(2^{-1} + 2^1)}{5.2^x 5^x}$$
$$= \frac{2^{-1} + 2^1}{5.5^x}$$
$$= \frac{\frac{1}{2} + 2}{5.5^x}$$
$$= \frac{\frac{5}{2}.5^{-x}}{5}$$
$$= \frac{\frac{5}{2}.10}{5}$$
$$= \frac{25}{5} = 5$$

1. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

2. Give learners an exercise to complete on their own.

3. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 1, Topic 2: Topic Overview

EQUATIONS AND INEQUALITIES

A. TOPIC OVERVIEW

- This topic is the second of four topics in Term 1.
- This topic runs for three weeks (13,5 hours).
- It is presented over 10 lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 13,5 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Equations and Inequalities is part of Algebraic expressions which counts 30% of the final Paper 1 examination.
- The ability to solve equations is a skill required in most topics in the curriculum.

Breakdown of topic into 10 lessons:

	Lesson title	Suggested time (hours)		Lesson title	Suggested time (hours)
1	Completing the square; maximum and minimum	2			
Solving quadratic equations:			6	Simultaneous equations	1.5
2	by completing the square	1	7	Word problems	1
3	by factorising	1	8	Quadratic inequalities	2
4	with fractions	1	9	Nature of roots	1.5
5	using the quadratic formula	1	10	Revision and consolidation	1.5

SEQUENTIAL TABLE

B

GRADE 10 and Senior phase	GRADE 11	GRADE 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
Solve: ● Linear equations ● Quadratic equations ● Literal equations ● Exponential equations ● Linear inequalities ● System of linear equations ● Word problems	● Completing the square Solve: ● Quadratic equations ● Quadratic inequalities ● Equations in two unknowns where one is linear, and one is quadratic ● (Exponential equations – covered in previous topic) ● Nature of roots	Solve: ● Logarithmic equations.

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

C

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Equations and Inequalities.

These include:

- A lack of understanding of roots
- Quadratic inequalities – integrating these with functions for a visual understanding is recommended.
- Factorising skills.

It is important that you keep these issues in mind when teaching this section.

While teaching Equations and Inequalities, always use the correct notation and mathematical language. Encourage learners to do the same. Encourage learners to check their answers to equations.

D

ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test with memorandum, and an investigation with a rubric are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). These assessments are provided in the Resource Pack – Resources 12 and 13.
- The questions usually take the form of algebraic expressions and fractions that need to simplified or factorised.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

E

MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
linear equation	An algebraic equation in which each term has an exponent of one and the graphing of the equation results in a straight line. When solved, there is only one possible solution
quadratic equation	An equation of the second degree, meaning it contains at least one term that is squared. The standard form is $ax^2 + bx + c = 0$ with a, b, and c are constants, or numerical coefficients, and x is an unknown variable. When solved, there are two possible solutions
literal equation	An equation with two or more variables
inequality	A mathematical sentence that uses symbols such as $<$, $\leq$, $>$ or $\geq$ to compare two quantities.
solution	The value of the variable that makes an equation true.
root	A real number x will be called a solution or a root if it satisfies the equation. The roots are the x-intercepts of a function, if the equation is drawn on a Cartesian plane.

formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value
like terms	Terms that have the same variables. For example: $2a$ and $4a$ are like terms and can be added or subtracted
inverse operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.
identity	An equation that is true for any values that replace the variable. That means the variable can be any real number
surd	A number or quantity that cannot be expressed as the ratio of two integers. It is an irrational number.

COMPLETING THE SQUARE; MAXIMUM AND MINIMUM

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	30

Lesson Objectives

By the end of the lesson, learners should be able to:

- complete the square on an expression
- determine maximum or minimum represented by the expression in the form $a(x-p)^2+q$.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 3 from the Resource Pack ready.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson have the three examples from point 1 ready.
6. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans. If you use one of the textbooks that don't have an exercise, source one from elsewhere. Learners need to practice the skill of completing the square before they need to use it to solve equations.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 2 3	22 23 26	1	24			2.1 – 2.7	24 – 31		

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Being able to confidently change an expression (and essentially a function for next term) from $ax^2 + bx + c$ to the form $a(x - p)^2 + q$ is a skill that will assist learners in both the solving of equations and Functions.

DIRECT INSTRUCTION

1. Write the following expressions on the board and ask learners to multiply out and simplify:

$(x - 3)^2 + 2$	$2(x + 1)^2 - 4$	$-3\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$

2. When learners have had enough time, do the solutions on the board with them.

$(x - 3)^2 + 2$ $= x^2 - 6x + 9 + 2$ $= x^2 - 6x + 11$	$2(x + 1)x^2 - 4$ $= 2(x^2 + 2x + 1) - 4$ $= 2x^2 + 4x + 2 - 4$ $= 2x^2 + 4x - 2$	$-3\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$ $= -3(x^2 + x + \frac{1}{2}) + \frac{11}{4}$ $= -3x^2 - 3x - \frac{3}{4} + \frac{11}{4}$ $=-3x^2 - 3x + 2$

3. Say: *Note how we have changed an expression in the format $a(x - p)^2 + q$ to a format of $ax^2 + bx + c$.*
   Write these on the board and say: *These are both quadratic expressions which also represent quadratic functions which will be covered in detail next term. Equations and functions have a direct relationship with each other.*

4. Say: *Changing from $(x - p)^2 + q$ to $ax^2 + bx + c$ requires a skill you have been practicing for many years – multiplying out then collecting like terms. Now we are going to learn how to change from $ax^2 + bx + c$ to $a(x - p)^2 + q$.*

5. Say: *This is called completing the square. Write this heading in your book.*

6. Before showing learners how to complete the square on an expression, remind them about perfect square trinomials.
   Write the following on the board and ask learners to factorise the trinomials:

$x^2 + 10x + 25$ $= (x + 5)(x + 5)$	$x^2 - 4x + 4$ $= (x - 2)(x - 2)$	$x^2 + 8x + 16$ $= (x + 4)(x + 4)$

7. Ask: *What do all the solutions have in common?*
   (The factors are repeated).
   Say: *Write each answer in a simpler way.*

$x^2 + 10x + 25$ $= (x + 5)(x + 5)$ $= (x + 5)^2$	$x^2 - 4x + 4$ $= (x - 2)(x - 2)$ $= (x - 2)^2$	$x^2 + 8x + 16$ $= (x + 4)(x + 4)$ $= (x + 4)^2$

8. Ask: What are these trinomials called?
   (Perfect square trinomials).

9. Use the three perfect square trinomials from above to show that:
   - The last term is a perfect square
   - The sign for the last term is always positive
   - The coefficient of the middle term is twice the square root of the last term OR half of the coefficient of the middle term when squared is equal to the last term

   (For example, in the first trinomial, half of 10 (5), when squared is equal to 25)
   Tell learners this is a key to understanding how to complete the square.

10. Do two fully worked examples with learners.
    Learners should write the examples in their books and make notes as they do so.

Example	Teaching notes
1. $x^2 - 6x - 12$	Ask: *is this a perfect square trinomial?* (No). Say: *We are going to use the concepts that we have discussed to change this trinomial into the format:* $a(x - p)^2 + q$.
$= x^2 - 6x$ _____ $- 12$ _____	Step 1: Get the expression 'ready' by rewriting it but leaving a space after the 2nd term We need to create a perfect square trinomial, so it can be factorised and form the $(x - p)^2$ part of the form we require.
$\left(\frac{-6}{2}\right)^2 = 9$	Step 2: Take the coefficient of x. Halve it and square it.
$= x^2 - 6x + 9 - 12 - 9$	Step 3: Add this after the first two terms AND subtract it. Remember: the expression cannot be a new one. It must remain equal to the original. Note the perfect square trinomial in front.
$= (x - 3)^2 - 12 - 9$	Step 4: Factorise the perfect square trinomial
$= (x - 3)^2 - 21$	Step 5: Collect like terms In the examples that follow, steps 4 and 5 will be done together.

2. $x^2 + 2x - 24$ $= x^2 + 2x$ ____ $- 24$ ____ $= x^2 + 2x + 1 - 24 - 1$ $= (x + 1)^2 - 25$	Note: allow learners to try some steps on their own. Ask: *What needs to be done next?* When learners have answered, tell them to complete the step before you do it on the board. Step 1: Get the expression 'ready' by rewriting it but leaving a space after the 2nd term Step 2: Take the coefficient of x. Halve it and square it. Step 3: Add this after the first two terms AND subtract it. Step 4: Factorise the perfect square trinomial and collect like terms.

11. Do another three fully worked examples with learners.
    Learners should write the examples in their books and make notes as they do so.

The following three examples have coefficients other than 1 in the 1st term. It is not impossible to complete the square if the coefficient of the 1st term is not 1. Therefore, taking out a factor will be necessary in the following examples. Note that the factor may not be common to all three terms.	
1. $3x^2 + 6x - 12$ $= 3[x^2 + 2x - 4]$ $= 3[x^2 + 2x$ __ $- 4$ __$]$ $= 3[x^2 + 2x + \mathbf{1} - 4 - \mathbf{1}]$ $= 3[(x + 1)^2 - 5]$ $= 3(x + 1)^2 - 15$	Step 1: Take out the coefficient of x^2 if it is not 1. Step 2: Get the expression 'ready' by rewriting it but leaving a space after the 2nd term Step 3: Take the coefficient of x. Halve it and square it. Step 4: Add this after the first two terms AND subtract it. Step 5: Factorise the perfect square trinomial and collect like terms. Step 6: Simplify by multiplying the factor taken out back in.
2. $2x^2 - 7x + 6$ $= 2\left[x^2 - \frac{7}{2}x + 3\right]$ $= 2\left[x^2 - \frac{7}{2}x\_\_ + 3\_\_\right]$ $\left(\frac{1}{2} \times -\frac{7}{2}\right)^2 = \frac{49}{16}$ $= 2\left[x^2 - \frac{7}{2}x + \frac{49}{16} + 3 - \frac{49}{16}\right]$ $= 2\left[\left(x - \frac{7}{4}\right)^2 - \frac{1}{16}\right]$ $= 2\left(x - \frac{7}{2}\right)^2 - \frac{1}{8}$	In this example learners need to work with fractions because the coefficient is not a factor of all the terms. Step 1: Take out the coefficient of x^2 if it is not 1. Step 2: Get the expression 'ready' by rewriting it but leaving a space after the 2nd term Step 3: Take the coefficient of x. Halve it and square it. Step 4: Add this after the first two terms AND subtract it. Step 5: Factorise the perfect square trinomial and collect like terms. Step 6: Simplify by multiplying the factor taken out back in.

3. $-x^2 + 5x - 3$ $= -[x^2 - 5x + 3]$ $= -[x^2 - 5x \_\_ + 3 \_\_]$ $\left(\frac{1}{2} \times -\frac{5}{2}\right)^2 = \frac{25}{4}$ $= -\left[x^2 - 5x + \frac{25}{4} + 3 - \frac{25}{4}\right]$ $= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{13}{4}\right]$ $= -\left(x - \frac{5}{2}\right)^2 + \frac{13}{4}$	Step 1: Take out the coefficient of x^2 if it is not 1 (–1 in this case) Step 2: Get the expression 'ready' by rewriting it but leaving a space after the 2nd term Step 3: Take the coefficient of x. Halve it and square it. Step 4: Add this after the first two term AND subtract it. Step 5: Factorise the perfect square trinomial and collect like terms. Step 6: Simplify by multiplying the factor taken out back in.

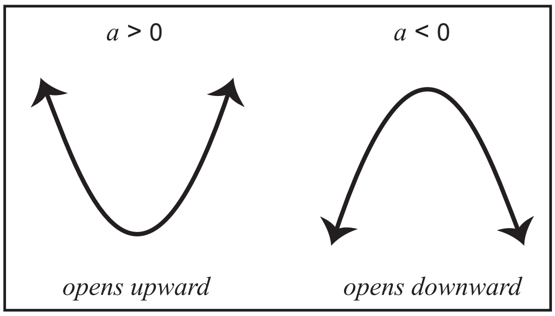
12. Ask learners if they have any questions. Give learners an exercise to do on completing the square on expressions before doing the maximum and minimum.

13. Once the exercise has been completed and corrected, move on to the second part of the lesson.

MAXIMUM AND MINIMUM

14. Tell learners that although they have not yet done functions this year, they will work with the quadratic function again. The expressions that they have just worked with are all quadratic functions and parabolas can be drawn from each one.

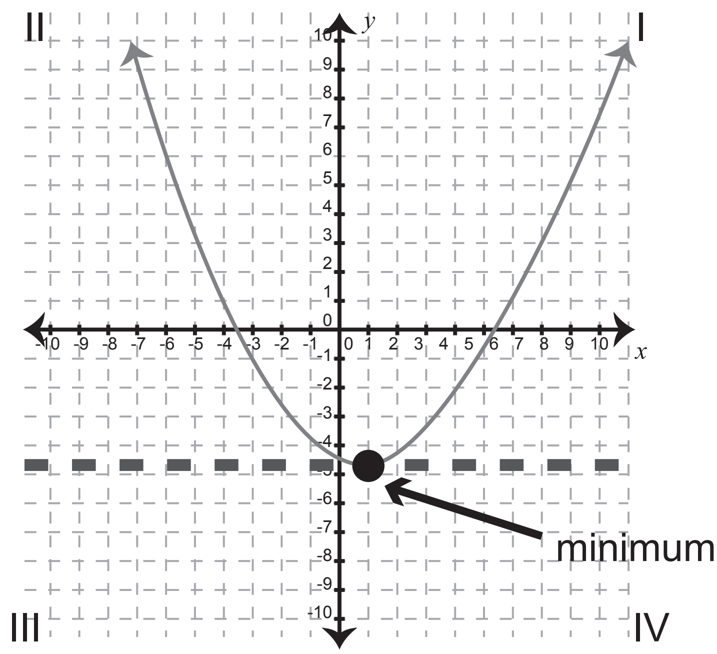
15. Say: *Come and draw a parabola on the board.*
    Once a parabola has been drawn, ask for a second to be drawn. Ask the question according to the one that was drawn first to ensure you have one that represents a parabola where $a > 0$ and one where $a < 0$.

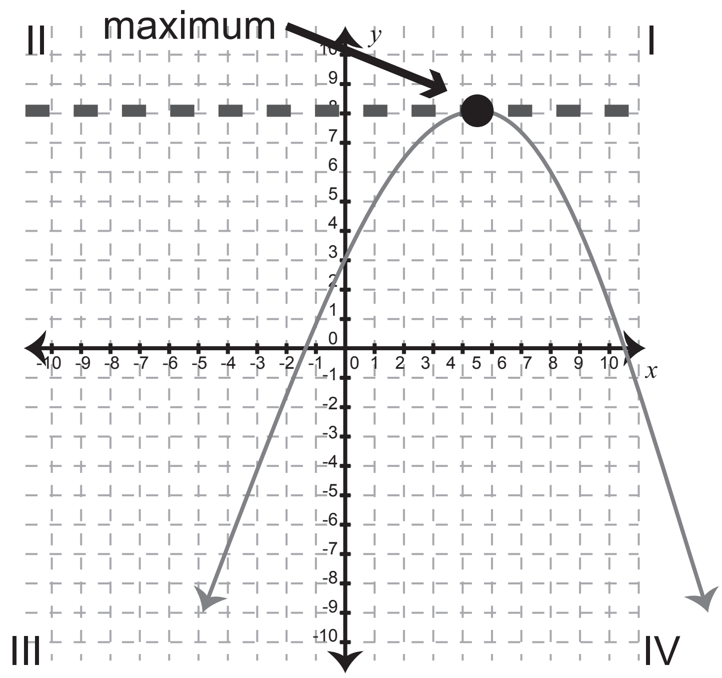


16. Discuss the turning point with learners. Fill in a reasonable turning point according to the sketch.

Note: learners will probably have drawn sketches where the turning point is also the y-intercept as that is what it looked like in Grade 10. Use these but after the explanation tell learners that in Grade 11 the turning point will not always be the same as the y-intercept. There is no need to spend too much time on this new idea as it will be covered in detail in Term 2. The maximum and minimum will also be covered again. Remember that the focus is on completing the square which will be used to solve equations.

17. Say: *Note how one parabola has a minimum value and the other has a maximum value* which have a direct connection to the turning point.
    Ask: *Which parabola has the minimum value? Which parabola has a maximum value?*





The diagrams above are available in the Resource Pack – Resource 3.

18. Write $a(x – p)^2 + q$ on the board. Tell learners that this format gives us the information about the parabola (quadratic function) that we have just looked at from a visual point of view.

If $a > 0$, the parabola will have a minimum value If $a < 0$, the parabola will have a maximum value	$(p;q)$ represents the turning point. The y-value of the turning point (q) gives us the maximum or minimum value of the function.
In the two diagrams above, the first parabola has a minimum vale of –4 and the second parabola has a maximum value of 8.	

19. Use the following examples to discuss minimum and maximum values and what they would be for each expression.

Expression:	Minimum or Maximum?	Value
$-(x - 2)^2 + 1$	Maximum (–1 < 0)	1
$(x + 3)^2 - 5$	Minimum (1 > 0)	–5
$3(x + 1)^2 - 10$	Minimum (3 > 0)	–10
$-2(x - 4)^2 + \frac{1}{8}$	Maximum (–2 < 0)	$\frac{1}{8}$

20. Ask directed questions so that you can ascertain learners' level of understanding.
Ask learners if they have any questions.

21. Give learners an exercise to complete on their own.

22. Walk around the classroom as learners do the exercise. Support learners where necessary

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=9lf6claQnYo

https://www.youtube.com/watch?v=ubs2gwTfcSY

https://www.youtube.com/watch?v=XScVP0UrN3M

TERM 1, TOPIC 2, LESSON 2

SOLVE QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Suggested lesson duration: 1 hour

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: ● solve quadratic equations by completing the square.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5	29	2	26	6	31	2.10	39	2.2	43

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners need to feel confident in their ability to complete the square on an expression before starting this lesson.

DIRECT INSTRUCTION

1. Say: *In Grade 11 almost all the equations you will solve will be quadratic equations.*
2. Ask: *What is a quadratic equation?*
   (An equation with two possible solutions).
3. Ask: *What is the common method for solving quadratic equations?*
   (Get all terms on one side equal to zero and factorise).
4. Say: *Some expressions cannot be factorised. We therefore need to find other methods to solve a quadratic equation. Today we are going to use the skill you learned in the previous lesson – completing the square.*
5. Write the following equation on the board:
   $x^2 + 6x - 12 = 0$
6. Point out that if learners had been given this equation in Grade 10 they would probably have thought it was not possible to solve as the expression on the left-hand side cannot be factorised. Today, however, they will learn a new method to find the solutions.

Tell learners at this stage that there are some equations that don't have real solutions and these will be discussed in a future lesson.

7. Before doing the example in full, point out that learners will not follow exactly the same steps that they used to complete the square on an expression and that they must take careful note where it may be a little different.
8. Say: *The main aim will be to get to a stage where the following type of statement will occur:*

   $$(x + 4)^2 = 20$$

   To solve for x, we would then find the square root of both sides using the inverse operation of squaring and subtract 4.

9. Say: *Note the perfect square. This is why we need to complete the square – to create a perfect square trinomial to factorise.*

10. Ask: *Where will the two solutions come from?*
(When we find the square root of a number there are two possible solutions – a positive and a negative solution).

11. Work through the examples with the learners.
Learners should write the examples in their books and make notes as they do so.

<table>
<tr><th>Example</th><th>Teaching notes</th></tr>
<tr><td colspan="2">Say: Because we need to form a perfect square trinomial on the left-hand side, we only need the first two terms there. The constant does not affect creating the perfect square trinomial. We can add or subtract the constant from both sides to ensure the constant is not in the way of creating a perfect square trinomial.
Note: this could also be done further on in the process, but experience has shown that most learners find it easier to deal with the constant first.</td></tr>
<tr><td>1.
$x^2 + 6x - 12 = 0$
$x^2 + 6x = 12$
$x^2 + 6x + 9 = 12 + 9$
$(x + 3)^2 = 21$
$\sqrt{(x+3)^2} = \pm\sqrt{21}$
$x + 3 = \pm\sqrt{21}$
$x = \pm\sqrt{21} - 3$</td><td>Step 1: Add or subtract the constant on both sides so the first two terms are the only terms on the left-hand side.
Step 2: Take the coefficient of x. Halve it and square it.
Step 3: Add this answer to BOTH sides (to keep the equation balanced).
Step 4: Factorise the perfect square trinomial and collect like terms on the right-hand side.
Step 5: Find the square root of both sides. Remember that there will be TWO solutions from performing this operation.
Step 6: Find the final two solutions by making the variable the subject of the equation.</td></tr>
<tr><td colspan="2">Ask: What differences did you notice compared to when you were completing the square on an expression?
(The constant had to be on the right-hand side; after halving and squaring the coefficient of the variable it got added to both sides – not added then immediately subtracted; because we were solving for the unknown variable, we had to use inverse operations to make the variable the subject of the equation).</td></tr>
</table>

2. $3x^2 - 6x - 6 = 0$ $3x^2 - 6x = 6$ $3(x^2 - 2x) = 6$ $x^2 - 2x = 2$ $x^2 - 2x + 1 = 2 + 1$ $(x - 1)^2 = 3$ $\sqrt{(x-1)^2} = \pm\sqrt{3}$ $x - 1 = \pm\sqrt{3}$ $x = \pm\sqrt{3} + 1$	Step 1: Add or subtract the constant on both sides so the first two terms are the only terms on the left-hand side. Step 2: Take the coefficient of x^2 out to enable us to complete the square. Step 3: Divide both sides of the equation by the factor taken out. Step 4: Take the coefficient of x. Halve it and square it. Step 3: Add this answer to BOTH sides (to keep the equation balanced). Step 5: Factorise the perfect square trinomial and collect like terms on the right-hand side. Step 6: Find the square root of both sides. Remember that there will be TWO solutions from performing this operation. Step 7: Find the final two solutions by making the variable the subject of the equation.
Ask: *What was done in this example that cannot be done if completing the square was done to an expression?* (We could divide both sides by the factor taken out because we were dealing with an equation; when dealing with an equation we can do 'anything' we need to providing the operation performed is done to both sides to keep the equation balanced and therefore true).	
3. $-2x^2 + 2x + 5 = 0$ $-2x^2 + 2x = -5$ $-2(x^2 - x) = -5$ $x^2 - x = \frac{5}{2}$ $x^2 - x + \frac{1}{4} = \frac{5}{2} + \frac{1}{4}$ $\left(x - \frac{1}{2}\right)^2 = \frac{11}{4}$ $\sqrt{\left(x - \frac{1}{2}\right)^2} = \pm\sqrt{\frac{11}{4}}$ $x - \frac{1}{2} = \pm\sqrt{\frac{11}{4}}$ $x = \frac{1}{2} \pm \sqrt{\frac{11}{4}}$ $x = \frac{1}{2} \pm \frac{\sqrt{11}}{\sqrt{4}}$ $x = \frac{1}{2} \pm \frac{\sqrt{11}}{2}$ $x = \frac{1 \pm \sqrt{11}}{2}$	Step 1: Add or subtract the constant on both sides so the first two terms are the only terms on the left-hand side. Step 2: Take the coefficient of x^2 out so that you can to complete the square. Step 3: Divide both sides of the equation by the factor taken out. Step 4: Take the coefficient of x. Halve it and square it. Step 3: Add this answer to BOTH sides (to keep the equation balanced). Step 5: Factorise the perfect square trinomial and collect like terms on the right-hand side. Step 6: Find the square root of both sides. Remember that there will be TWO solutions from performing this operation. Step 7: Find the final two solutions by making the variable the subject of the equation. Note that this can be simplified further. 4 is a perfect square. Surd rules can be used. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

11. Say: *Sometimes a question may give the instruction to solve the equation and round to two decimal places. The solutions in the above examples are in simplest surd form.*

12. Learners should use their calculators and to find the solutions of the examples done correct to two decimal places.

$x = \pm\sqrt{21} - 3$ $x = 1{,}58$ or $x = -7{,}58$	$x = \pm\sqrt{3} + 1$ $x = 2{,}73$ or $x = -0{,}73$	$x = \frac{1 \pm \sqrt{11}}{2}$ $x = 2{,}16$ or $x = -1{,}16$

13. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

14. Give learners an exercise to complete on their own.

15. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=prx_Bf2hakw

https://www.youtube.com/watch?v=5iH2V8nboZg

https://www.youtube.com/watch?v=DJMH2F3GuIc

SOLVING QUADRATIC EQUATIONS BY FACTORISING

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: • solve quadratic equations by factorising.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4	27	3 (1-16)	30	1 5	23 29	2.8	34	2.1 (1-18) 2.4	38 39

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have been solving quadratic equations by factorising since Grade 9.
2. There are no new factorising concepts, but learners factorise and solve more complicated equations in Grade 11 and sometimes need to use other skills.
3. In this lesson learners revise quadratic equations that can easily be factorised as well as the use of substitution to assist in solving more complicated equations.

DIRECT INSTRUCTION

1. Ask learners to solve the following six equations. As they work, walk around the classroom and assist where necessary. Once learners have had the chance to complete the questions, correct them in full on the board. Point out issues that you noticed while assisting learners as they were working.

1	$(2x - 1)(x + 3) = 0$	2	$y(3y + 4) = 0$
3	$5x^2 = 8x$	4	$2a(a - 1) + 3(a - 1) = 0$
5	$2x^2 + 3x = 27$	6	$2(x - 2)(x + 2) + 6x = (x - 1)^2$

Solutions and notes:

<table>
<tr><td>1.
$(2x - 1)(x + 3) = 0$
$2x - 1 = 0$ or $x + 3 = 0$
$2x = 1 \qquad x = -3$
$x = \frac{1}{2}$</td><td rowspan="2">Both equations are already factorised and equal to zero and are therefore ready to be solved.
Many learners make the mistake of multiplying out first and trying to factorise again (and sadly make mistakes at this stage).</td></tr>
<tr><td>2.
$y(3y + 4) = 0$
$y = 0$ or $3y + 4 = 0$
$3y = -4$
$y = -\frac{4}{3}$</td></tr>
</table>

3. $5x^2 = 8x$ $5x^2 - 8x = 0$ $x(5x - 8) = 0$ $x = 0$ or $5x - 8 = 0$ $5x = 8$ $x = \frac{8}{5}$	Remind learners that when solving a quadratic equation, all terms need to be on one side and equal to zero before factorising.
4. $2a(a - 1) + 3(a - 1) = 0$ $(a - 1)(2a + 3) = 0$ $a - 1 = 0$ or $2a + 3 = 0$ $a = 1$ $\quad 2a = -3$ $a = -\frac{3}{2}$	Learners may need to be reminded how to factorise by grouping. There are two terms here with a common factor of $a - 1$.
5. $2x^2 + 3x = 27$ $2x^2 + 3x - 27 = 0$ $(2x + 9)(x - 3) = 0$ $2x + 9 = 0$ or $x - 3 = 0$ $2x = -9$ $\quad x = 3$ $x = -\frac{9}{2}$	Remind learners that when solving a quadratic equation, all terms need to be on one side and equal to zero before factorising.
6. $2(x - 2)(x + 2) + 6x = (x - 1)^2$ $2(x^2 - 4) + 6x = x^2 - 2x + 1$ $2x^2 - 8 + 6x = x^2 - 2x + 1$ $x^2 + 8x - 9 = 0$ $(x + 9)(x - 1) = 0$ $x + 9 = 0$ or $x - 1 = 0$ $x = -9$ $\quad x = 1$	This question requires multiplying out, collecting like terms and then getting all the terms on one side before factorising.

2. Ask learners if they have any questions. Give learners an exercise to complete on their own.
3. Once learners have completed the exercise and it has been corrected, move on to the next concept.

USING SUBSTITUTION TO SOLVE MORE COMPLICATED EQUATIONS

4. Say: *Instead of working with complex variables, it is often easier to simplify an equation by using substitution then completing the solution afterwards.*

5. Write the following equation on the board:

$$2(x + 5)^2 + 3(x + 5) - 2 = 0$$

6. Say: *Although we could multiply out and collect like terms and then factorise, we will use this example to demonstrate another method.*

7. Say: *Note that the left-hand side is a trinomial.* Ask: *Do you think this equation would be easier to solve if it looked like this:* $2k^2 + 3k - 2 = 0$*?*

8. Instruct learners to solve this equation.

$$\begin{aligned} 2k^2 + 3k - 2 &= 0 \\ (2k - 1)(k + 2) &= 0 \\ 2k - 1 = 0 \quad &\text{or} \quad k + 2 = 0 \\ 2k = 1 \quad &\qquad k = -2 \\ k = \frac{1}{2} & \end{aligned}$$

9. Say: *Although we have solved the equation that looks like the original one, it isn't exactly the same – we haven't solved the original equation. We need to connect the two.*

10. Start again and show learners how to solve the original equation by using substitution. Point out that this method works when an expression is repeated in the equation.

11. Say: *Let's start again:*

Solve for x	Teaching notes
$2(x + 5)^2 + 3(x + 5) – 2 = 0$ Let $x + 5 = k$ $2k^2 + 3k – 2 = 0$ $(2k – 1)(k + 2) = 0$ $2k – 1 = 0$ or $k + 2 = 0$ $2k = 1$ $\quad k = –2$ $k = \frac{1}{2}$ $\therefore\ x + 5 = \frac{1}{2}$ or $x + 5 = –2$ $x = -4\frac{1}{2}$ $\quad x = –7$	Ask: *What is the repeated expression in this equation?* $(x + 5)$ Say: *To make this equation easier to solve, we can choose a variable that could replace the repeated expression.* Say: *Let the repeated expression equal k, then re-write the expression and solve the equation.* Once the equation has been solved with the aid of the substitution, the solutions need to be linked back to the substitution made and solve for x.
$\frac{1}{x^2 - x - 1} = x^2 – x – 1$ Let $x^2 – x – 1 = k$ $\frac{1}{k} = k$ LCD $= k$ $1 = k^2$ $0 = k^2 – 1$ $0 = (k + 1)(k – 1)$ $k = –1$ or $k = 1$ $\therefore\ x^2 – x – 1 = –1$ or $x^2 – x – 1 = 1$ $x^2 – x = 0$ $\quad x^2 – x – 2 = 0$ $x(x – 1) = 0$ $\quad (x – 2)(x + 1) = 0$ $x = 0$ or $x = 1$ or $x = 2$ or $x = –1$	Ask: *What is the repeated expression in this equation?* $(x^2 – x – 1)$ Say: *To make this equation easier to solve, we can choose a variable that could replace the repeated expression.* Tell learners to let the repeated expression equal k, then re-write the expression and solve the equation. Ask: *what should we do when solving equations with fractions in them?* (find the LCD to multiply through) Once the equation using k has been solved, ask: *what do we still need to do?* (replace k with the expression, $x^2 – x – 1$ and solve the two new equations)

Ask learners to spend a few minutes looking at the original equation. Ask: *Why are there four solutions to this equation?* (If, at the beginning, the LCD of $2(x^2 - 2x)^2$ was used to multiply throughout, an equation in the 4^{th} degree would be produced.)	
$3^{2x} - 10.3^x + 9 = 0$ Let $k = 3^x$ $k^2 - 10k + 9 = 0$ $(k - 9)(k - 1) = 0$ $k = 9$ or $k = 1$ $\therefore 3^x = 9$ or $3^x = 1$ $3^x = 3^2$ $3^x = 3^0$ $\therefore x = 2$ or $x = 0$	Point out that 3^{2x} could be written as $(3^x)^2$. Ask: *How does this help us use our new method of solving equations?* (3^x is a repeated expression and can be replaced by k to make the equation easier to solve). Once the equation using k has been solved, ask: What do we still need to do? (Replace k with 3^x and solve the two new equations).

12. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

13. Give learners an exercise to complete on their own. If the textbook, you use doesn't have an exercise on this you need to source some questions from elsewhere.

14. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=nXd6WQeW8H8

SOLVING QUADRATIC EQUATIONS WITH FRACTIONS

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: • solve quadratic equations with fractions.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
7	32	3 (17-24)	30	2	25	2.9	36	2.1 (19-end)	38

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This is an extension of the previous lesson. Learners will still use factorisation to solve equations, but the equations will have fractions in them.

DIRECT INSTRUCTION

1. Say: *The equations that we will solve today will all have fractions in them.*
2. Say: *You may recall that we dealt briefly with a fraction in an equation in the previous lesson. You have also dealt with fractions in equations in previous years.*
3. Ask: *What should be the first step to deal with fractions in equations?*
   (Find the lowest common denominator and multiply each term with it to remove all fractions).
4. Say: This is true. However, sometimes the expressions in the denominators are not in their simplest form and may need to be simplified.
5. Another important aspect is to look carefully at the denominator. If a variable is involved, this means that there must be restrictions on the variable.
   Ask: *What can there never be in the denominator?*
   (Zero).
6. Write these expressions on the board:

x	$x + 2$	$2x - 5$

Ask: If these expressions were in the denominator position, what would the restriction on the variable be?

$x \neq 0$	$x + 2 \neq 0$ $\therefore x \neq -2$	$2x - 5 \neq 0$ $2x \neq 5$ $x \neq \frac{5}{2}$

7. Do a few examples with learners.
   Learners should write the examples in their books and make notes as they do so.

Example	Teaching notes
1. $\frac{10}{3(x-1)} + \frac{1}{3} = \frac{4}{x+2}$	Point out that each of the denominators is in simplest form. This means the LCD can be found and multiplied through, but first the restrictions must be stated. Once the LCD has been used to multiply through, tell learners they need to multiply out and solve the equation as usual. Allow learners to complete the example on their own after helping them get to this step. Remind learners to check that the solutions are valid by checking the restrictions.
$\frac{10}{3(x-1)} + \frac{1}{3} = \frac{4}{x+2}$ $x \neq 1\ ;\ –2$ LCD = $3(x – 1)(x + 2)$ $3(x-1)(x+2) \times \frac{10}{3(x-1)} + 3(x-1)(x+2) \times \frac{1}{3} = \frac{4}{x+2} \times 3(x-1)(x+2)$ $10(x + 2) + (x – 1)(x + 2) = 12(x – 1)$ $10x + 20 + x^2 + x – 2 = 12x – 12$ $x^2 – x + 30 = 0$ $(x + 6)(x – 5) = 0$ $x = –6$ or $x = 5$	
2. $\frac{x}{2x-4} - \frac{x}{x-2} = 1$	Point out that the denominator in the first fraction needs to be factorised before the LCD can be found. Once that has been completed, follow the steps in Example 1.
$\frac{x}{2(x-2)} - \frac{x}{x-2} = 1$ $x \neq 2$ LCD $= 2(x – 2)$ $2(x-2) \times \frac{x}{2(x-2)} - 2(x-2) \times \frac{x}{x-2} = 1 \times 2(x-2)$ $x – 2x = 2x – 4$ $–3x = –4$ $x = \frac{4}{3}$	

<table>
<tr>
<td>3.
$$\frac{2}{x-2} = \frac{3x+6}{x^2-x-2} + \frac{2}{x^2+4x+3}$$</td>
<td>Point out that the denominator in the first fraction needs to be factorised before the LCD can be found. Once that has been completed, follow the steps in Example 1.</td>
</tr>
<tr>
<td colspan="2">

$$\frac{2}{x-2} = \frac{3x+6}{(x-2)(x+1)} + \frac{2}{(x+3)(x+1)}$$

$$x \neq 2;\ -1;\ -3$$

LCD $= (x-2)(x+1)(x+3)$

$$(x-2)(x+1)(x+3) \times \frac{2}{x-2} = (x-2)(x+1)(x+3) \times \frac{3x+6}{(x-2)(x+1)} + \frac{2}{(x+3)(x+1)} \times (x-2)(x+1)(x+3)$$

$$2(x+1)(x+3) = (x+3)(3x+6) + 2(x-2)$$

$$2(x^2+4x+3) = 3x^2+15x+2x-4$$

$$2x^2+8x+6 = 3x^2+15x+2x-4$$

$$-x^2-9x+10 = 0$$

$$x^2+9x-10 = 0$$

$$(x+10)(x-1) = 0$$

$$x = -10 \quad \text{or} \quad x = 1$$

</td>
</tr>
</table>

8. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

9. Give learners an exercise to complete on their own.

10. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=rqm3Ms-11BU

TERM 1, TOPIC 2, LESSON 5

SOLVING QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA

Suggested lesson duration: 1 hour

A

POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: • solve quadratic equations by using the quadratic formula.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
6	31	4 5	31 31	7	33	2.11	42	2.3 2.6	47 52

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This will be the second method that learners will use to solve equations with expressions that cannot be factorised.

DIRECT INSTRUCTION

1. Remind learners how they solved equations by completing the square earlier on in this topic.
2. Ask: *Why did we require another method to solve quadratic equations?*
   (The expressions in the equations would not factorise).
3. Say: *Today we are going to learn another method to solve equations when faced with the same problem. It involves a formula. To use the formula, the quadratic expression must be in standard form.*
4. Ask: *What is the standard form of a quadratic expression?*

   ($ax^2 + bx + c$)
5. In other words, a is the coefficient of x^2, b is the coefficient of x and the c is the constant.
6. Write the quadratic formula on the board:
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
7. Say: *Substitution is a skill required for this.*
8. Ask: *We know that a quadratic equation has two solutions. Which part of this formula will produce two solutions?*
   (±).

9. Tell learners that they will first practice solving a few equations using the quadratic formula before you show them how it was derived.
Learners should write the examples in their books and make notes as they do so.

Example	Teaching notes
1. $x^2 + 6x - 2 = 0$ $a = 1 \quad b = 6 \quad c = -2$ $x = \frac{-b\sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-2)}}{2(1)}$ $x = -3 \pm \sqrt{11}$ $x = 0{,}32$ or $x = -6{,}32$	Say: *Write the values for a, b and c then use them to substitute into the quadratic formula.* Say: *If the instruction in a question only says 'solve' then this answer is acceptable. This answer is also in simplest surd form.* Should the instruction be to give your answer to two decimal places, then one more step on the calculator is required.
2. $5x^2 - \frac{1}{4}x = 3$ $5x^2 - \frac{1}{4}x - 3 = 0$ $a = 5 \quad b = -\frac{1}{4} \quad c = -3$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-\left(-\frac{1}{4}\right) \pm \sqrt{\left(-\frac{1}{4}\right)^2 - 4(5)(-3)}}{2(5)}$ $x = \frac{3}{4}$ or $x = \frac{4}{5}$	Ask: *Is this in standard form?* (This needs to be rewritten before listing the values of a, b and c). Learners could multiply all terms by 4 (the LCD) if they wanted to but point out to them that they would get the same answers. From this step, the equation will be solved the same as above.
3. $7(x - 3)(x + 2) = 6x - 2$ $7(x^2 - x - 6) = 6x - 2$ $7x^2 - 7x - 42 = 6x - 2$ $7x^2 - 13x - 40 = 0$ $a = 7 \quad b = -13 \quad c = -40$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(7)(-40)}}{2(7)}$ $x = 3{,}49$ or $x = -1{,}64$	Ask: *Is this in standard form?* (Each side needs to be multiplied out and simplified then all terms need to be on one side equal to zero before listing the values of a, b and c). From this step, the equation will be solved the same as above.

10. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

11. Give learners an exercise to complete on their own.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

13. Once the exercise has been completed and corrected, tell learners that you are going to show them how the formula was derived.

14. Ask: *What is the standard form of a quadratic equation?*

    ($ax^2 + bx + c = 0$)

15. Say: *We are going to complete the square on this expression. You need not write it down yet, rather focus on listening and understanding.* Encourage learners to ask questions if they feel confused or lost and repeat any steps where necessary.

	Steps
$ax^2 + bx + c = 0$	Subtract the constant from both sides
$ax^2 + bx = -c$	Take out the coefficient of x^2
$a(x^2 + \frac{b}{a}x) = -c$	Divide both sides by a
$\frac{a\left(x^2 + \frac{b}{a}x\right)}{a} = \frac{-c}{a}$	Simplify
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Prepare the expression on the left-hand side to complete the square; Halve the coefficient of x and square it; Add this to both sides
$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{c}{a} + \frac{b^2}{4a^2}$	Working: $\left(\frac{1}{2} \times \frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ Factorise the perfect square trinomial
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	Use the rules of fractions to simplify the right-hand side
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Find the square root of both sides to solve for x

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$ $x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$ $x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	

16. Tell learners that they can copy the example down now. Ask questions as necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=3ayhvAI3IeY

https://www.youtube.com/watch?v=JSwjmTFMDwg

https://www.youtube.com/watch?v=H5AM1bzqCQw

TERM 1, TOPIC 2, LESSON 6

SIMULTANEOUS EQUATIONS

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: • solve simultaneous equations where one is quadratic and the other is linear.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have resources 4, 5 and 6 from the Resource Pack ready for use in the lesson.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson draw the sketches from point 1 on the board.
6. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
9	41	11 12	40 41	10	37	2.14 2.15	48 50	2.9	73

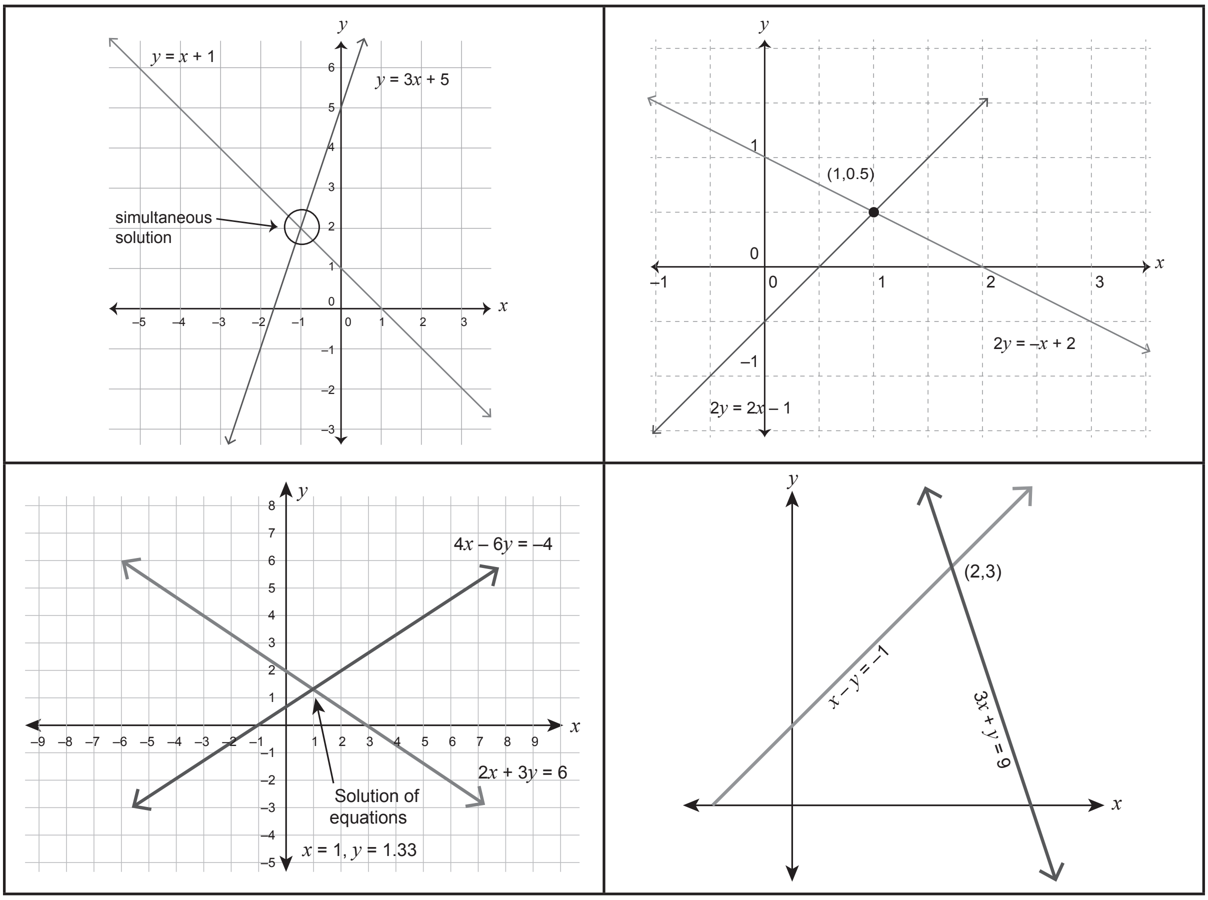
C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have already solved simultaneous equations where both equations are linear.

DIRECT INSTRUCTION

1. Draw a few sketches of straight lines intersecting on the board like these:
   These are available in the Resource Pack. Resources 4 and 5.



2. Say: *When you have solved simultaneous equations in the past, you always solved linear equations. These produced one x-value and one y-value.*

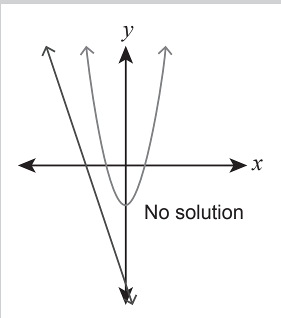
3. Ask: *Why is this?*
   (There is only one point of intersection).

4. Do an example in full to revise the basic concepts of solving simultaneous equations. Learners should write the example in their books and make notes as they do so.

Example	Teaching notes
Solve the following equations simultaneously: $2y = 2x - 1$ and $2y + x - 2 = 0$ $2y + x - 2 = 0$ $x = 2 - 2y$ $2y = 2x - 1$ $2y = 2(2 - 2y) - 1$ $2y = 4 - 4y - 1$ $6y = 3$ $y = \frac{1}{2}$ $x = 2 - 2y$ $x = 2 - 2\left(\frac{1}{2}\right)$ $x = 1$	Remind learners of the steps to follow: ● Get ONE of the variables by itself in ONE of the equations. ● Use this information to substitute into the second equation. You should now have an equation with only one unknown variable. ● Solve for this variable. ● Substitute the variable found back into the first equation and solve for the second variable.

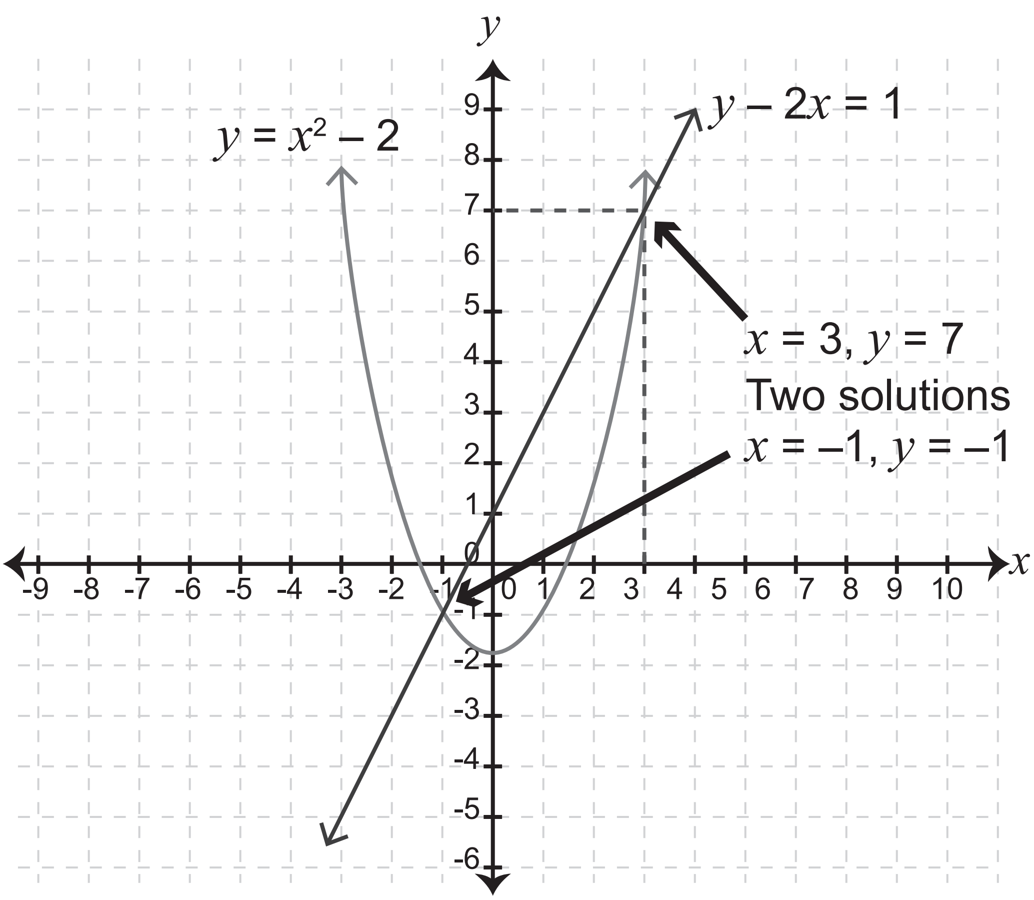
5. Ask: *What would be the result if we solved simultaneous equations where one equation was quadratic, and one equation was linear?*
   (There would be two solutions because a parabola and a straight line would have two points of intersection).

Although this statement is true, point out that a hyperbola and a straight line would also have two solutions – this would also become a quadratic once substitution has taken place. It may be a good idea to point out that there won't necessarily always be two solutions. This would be easy to see using the graphs. For example,



6. Say: *The steps to solving simultaneous equations where one equation is quadratic and the other is linear follows the same steps as when the equations are both linear.*

7. Do two examples with learners now.
   Learners should write the examples in their books and make notes as they do so.

<table>
<tr><th>Examples
Solve simultaneously:</th><th>Teaching notes</th></tr>
<tr><td>1.
$y = x^2 - 2$ and $y - 2x = 1$
$y - 2x = 1$
$x^2 - 2 - 2x = 1$
$x^2 - 2 - 2x - 1 = 0$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3$ or $x = -1$
$y - 2x = 1$
$y - 2(3) = 1$ $y - 2(-1) = 1$
$y = 1 + 6$ $y = 1 - 2$
$y = 7$ $y = -1$</td><td>Remind learners of the steps to follow:
• Get ONE of the variables by itself in ONE of the equations.
• Use this information to substitute into the second equation. You should now have an equation with only one unknown variable in (a quadratic equation).
• Solve for this variable.
• Substitute the two solutions found back into the first equation and solve for the second variable to find the corresponding values.</td></tr>
<tr><td colspan="2">The solutions above are represented graphically:


(This sketch is available in the Resource Pack – Resource 6).</td></tr>
</table>

2.

$$y = 3x + 7 \text{ and } y = 2x^2 + 8$$

$$y = 2x^2 + 8$$

$$3x + 7 = 2x^2 + 8$$

$$0 = 2x^2 - 3x + 1$$

$$0 = (2x - 1)(x - 1)$$

$$2x - 1 = 0 \quad \text{or} \quad x - 1 = 0$$

$$2x = 1 \qquad\qquad x = 1$$

$$x = \frac{1}{2}$$

$$y = 3\frac{1}{2} + 7 \qquad\qquad y = 3(1) + 7$$

$$y = \frac{17}{2} \qquad\qquad y = 10$$

8. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

9. Give learners an exercise to complete on their own.

10. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=6mNhX6VAaBM&list=PLOaNAKtW5HLTSR0eRjIgPOD6HoS-BoF07W&index=5&t=0s

TERM 1, TOPIC 2, LESSON 7

WORD PROBLEMS

Suggested lesson duration: 1 hour

A

POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: ● solve word problems using knowledge of equations.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson have the list of points from point 3 ready.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
10	44	7	34	11	39	2.17 2.18	65 67	2.10	79

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. In general, learners struggle with word problems. The only way this can change is for them to practice as many as possible on a regular basis.
2. It is not possible to show learners every possible word problem they will ever encounter. This lesson will be like the lesson on word problems in Grade 10. Various types of word problems will be discussed and an example done of each.
3. Rate will be added to the list for Grade 11.
4. Solving problems is an integral part of life and an important skill to have.

DIRECT INSTRUCTION

1. Start the lesson by telling learners how important it is to have the ability to solve a problem.
2. Tell them that you know this can be difficult. In this lesson you will give them as many tools as possible to help them become more confident in solving word problems. There are a few different types of problems which are more common than others at this level. These will be looked at one at a time.

> Note: Throughout the lesson, remind learners that the tips and steps you give them are not hard and fast rules. If they can solve any problem using a different strategy that is a positive sign - they must not feel that they must do it a certain way.

3. Start with some general guidelines for solving word problems.
   Learners should write these in their books.

> 1. Read each problem three times. The first reading is to determine what the problem is, the second to identify the "maths" words and the third is to set up an equation.
> 2. Decide what exactly is being asked and make this the variable ready to be represented in the equation. If there are two unknowns, make the smaller one the variable chosen.
> 3. If two or more items are involved, express one item in terms of the other.
>   For example: The boy is twice as old as his sister. The sister is younger, so she can be represented by the variable chosen (x being the most common) and the boy's age will then be $2x$.
> 4. Set up your equation using any other information given in the statement.
> 5. Solve the equation.
> 6. Answer the question asked.
> 7. Make sure the answer makes sense. For example, age cannot be negative.

4. Say: We are going to look at six different types of questions and do an example for each one. Learners should write the examples in their books and make notes as they do so.

NUMBER QUESTIONS

5. Discuss some general points regarding numbers.
   Ask: *If a number is x, what will the next consecutive number be?*
   $(x + 1)$
   Ask: *If an even number is x, what will the next even number be?*
   $(x + 2)$
   Ask: *If an odd number is x, what will the next odd number be?*
   ($(x + 2)$ – odd numbers are also 2 digits apart)

6. Do this example on the board:

The product of two consecutive integers is 72. Find the integers.	
$\therefore\ x(x + 1) = 72$ Solve the equation: $x(x + 1) = 72$ $x^2 + x - 72 = 0$ $(x + 9)(x - 8) = 0$ $x = -9$ or $x = 8$ If $x = -9$ $x + 1$ will equal -8 If $x = 8$ $x + 1$ will equal 9 Answer the question: The two numbers are either –9 and –8 OR 8 and 9.	Ask: *What is being asked?* (What the two integers are). Let the first number be x. Ask: *What will the second number be, considering that the first number is x?* $(x + 1)$ These two numbers must multiply together to make 72 Make the equation using this information and solve it.

AGE QUESTIONS

7. Share the following information with learners:
   A table is always useful in these types of questions. There is often some information about now and other information about some time in the past or the future. Now and the other time mentioned will be the columns in the heading and the two people involved will be the headings for the rows. If there is no comparison with now and another time, a table is not necessary.

8. Do the following example on the board:

Sara's father is six times as old as Sara. The product of their ages is 150 years. What are their respective ages?

Ask: *What is being asked?* (The ages of Sara and her father). Say: *It is always easier to make the younger person's age be x.* Let Sara's age be x. Ask: *How can we represent the father's age, considering that the Sara's age is x?* ($6x$) Say: *Now we need to make an equation.*
The product of their ages is 150 $x \times 6x = 150$ $6x^2 = 150$ $6x^2 - 150 = 0$ $6(x^2 - 25) = 0$ $x^2 - 25 = 0$ $(x + 5)(x - 5) = 0$ $x = -5$ or $x = 5$ Sara cannot be a negative age. Sara is 5 years old and her father is 6(5) = 30 years old.

SHOPPING

9. Say: *It is often useful to draw up a table in this type of question.*

10. Do this example with learners:

My mother bought 12m of material. Some of the material came from a roll with a flaw in it and only cost R20/m, instead of the normal price of R30/m. If mother paid R320 in total, how many metres of the damaged material did she buy?
Ask: *What is being asked?* (How many metres of damaged material was bought). Let this be x Ask: *If 12m were bought altogether and xm of the damaged material was bought, how many metres of the undamaged material were bought?* ($12 - x$) Note: If learners struggle with this, use actual numbers for a few questions then ask how they found the answer. For example: *If the mother bought 3m of damaged material how much undamaged material did she buy?* (9m because 12 subtract 3 is 9).

Draw up a table with the two types of material in the rows, and columns for the number of metres, the price per metre and the total cost.
Once the x and $(12 – x)$ have been filled in, it is possible to populate the rest of the table.
Once the table is complete, discuss what equation is possible to solve for the variable.
Ask: *What information was given that hasn't been used yet?*
(The total amount spent).
Say: *The total cost column would equal the total amount spent.*

	Number of metres	Cost per metre	Total cost
Damaged material	x	R20	$20x$
Undamaged material	$12 - x$	R30	$30(12 - x)$

$$20x + 30(12 - x) = 320$$
$$20x + 360 - 30x = 320$$
$$-10x = -40$$
$$x = 4$$

Mother bought 4m of damaged material.

MEASUREMENT

11. Say: *It is sometimes useful to make a sketch in this type of question.*

12. Do the following example on the board:

A small rectangular vegetable garden is enlarged by increasing the length by 3m and the width by 1m. The area of the new garden is three times larger than that of the original garden. Determine the original dimensions of the garden, if its area was $6m^2$.

Ask: *What is being asked?*
(The dimensions of the original vegetable garden).
Let these be x and y.
Ask: *What statement can be made about the original dimensions?*
($xy = 6$).
Say: *now let us consider the new garden>*
Ask: *What are the new dimensions?*
($x + 3$ and $y + 1$).
Ask: *What statement can be made about these dimensions?*
$(x + 3)(y + 1) = 3(6)$
Ask: *What concept we will be using to solve for x and y?*
(Simultaneous equations).

$xy = 6$ and $(x + 3)(y + 1) = 18$

$xy = 6$

$y = \frac{6}{x}$

$(x + 3)(y + 1) = 18$

$(x + 3)\left(\frac{6}{x} + 1\right) = 18$

$6 + x + \frac{18}{x} + 3 = 18$ LCD $= x$

$6x + x^2 + 18 + 3x = 18x$

$x^2 - 9x + 18 = 0$

$(x - 6)(x - 3) = 0$

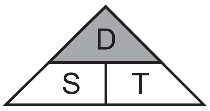
$x = 6$ or $x = 3$

$\therefore y = 1$ or $y = 2$

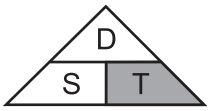
The original dimensions were 6m and 1m or 3m and 2m.

SPEED, DISTANCE AND TIME

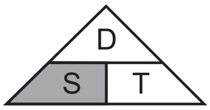
13. Learners need to know the relationship between distance, speed and time. Some learners like to use the triangle to help them:



Distance = Speed x Time



Time = $\frac{\text{Distance}}{\text{Speed}}$



Speed = $\frac{\text{Distance}}{\text{Time}}$

Show learners that if they cover the aspect they are looking for (here it is shaded), the formula required will be clear.

14. Tell learners that a table is a good tool for these questions as well.

15. Do the following example on the board:

Two marathon runners set off at 6h00 in opposite directions. One runs at an average speed of 12km/h and the other runs at an average speed of 8km/h.
At what time will they be 90km apart?

Say: *First draw up a table with two rows to represent the two runners and three columns for speed, distance and time.*
Ask: *What is being asked?*
(Time – hours).
Let the time (number of hours) be x.
Ask: *Will both runners' times be the same?*
(Yes – they will be 90km apart at the same time even though one runner will have run further).
Say: *Fill in the time (x) on the table for each runner.*
Ask: *Do we know anything else for sure that we can put on the table?*
(Yes – the speed of the two runners).
Tell learners to fill the speed in on the table for each runner.

This is an important part of the process. Point out that we don't know the distance – only that the distance of each runner combined will add to 90km.
Point out that we do know a formula for distance once we have a value for speed and time.
Ask: *How do we find distance?*
(speed × time).
Say: *Multiply each runner's speed by his time and fill it in under distance.*

Once the table has been populated, ask: What information has not yet been used?
(The 90km).
Say: *This will be used to form an equation and solve for x.*

	speed	**distance**	**time**
Runner 1	12km/h	$12 \times x$	x
Runner 2	8km/h	$8 \times x$	x

$$12x + 8x = 90$$
$$20x = 90$$
$$\frac{20x}{20} = \frac{90}{20}$$
$$x = 4{,}5$$

It will take 4,5 hours for them to be 90km apart
∴ they will be 90km apart at 11h30.

RATE

16. This is probably a new type of question for learners.

17. Rate of flow problems deal with objects or people completing a specific task in a certain time. In general, the problems work with inverse proportion as the more objects or people you add, the less time a task will take to complete.

18. Give learners the following points to write down about these types of questions:

How to recognise a rate problem	How to solve a rate problem
• You are given how long it takes to do a job in one way. • You are told how long it takes to do the same job in another way. • You are asked to find how long it would take to do the job combining the two ways.	• Identify the time unit. • Write down how much of the job the first person (or machine) could do in one unit of time. • Write down how much of the job the second person (or machine) could do in one unit of time. • Add these two answers together and make them equal to $\frac{1}{x}$ (where x represents the total time combined).

19. Do the following example with learners:

Mr Ndlovu, one of the Grade 11 mathematics teachers at your school, can mark a set of tests in 4 hours. Mrs Buswayo, another mathematics teacher, can mark a set of tests in 3 hours. How long would it take them to mark a set of tests if they worked together?
Ask: *What is the time unit?* (Hours). Ask: *How much of the job (marking a set of tests) can Mr Ndlovu do in one hour?* $\left(\frac{1}{4}\right)$. Ask: *How much of the job (marking a set of tests) can Mrs Buswayo do in one hour?* $\left(\frac{1}{3}\right)$ Say: *Let the total time to do the job together be* x *As we have represented how much of the job the teachers will be doing in one hour, this will be represented in the same way:* $\frac{1}{x}$

$$\frac{1}{4}+\frac{1}{3}=\frac{1}{x}$$

$$\text{LCD} = 12x$$

$$\frac{12x}{1}\times\frac{1}{4}+\frac{12x}{1}\times\frac{1}{3}=\frac{1}{x}\times\frac{12x}{1}$$

$$3x+4x=12$$

$$7x=12$$

$$x=\frac{12}{7}$$

$\approx$1,72hours (which is almost 1 hour 45 minutes).

20. Remind learners that the types of questions and examples you have just done with them by no means covers anything they could ever be asked. These examples are merely a guide to assist them. They should practice as many as possible. The more examples they try, the more confident they will become.
21. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
22. Give learners an exercise to complete with a partner.
23. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=Mps38dhHXkc

https://www.youtube.com/watch?v=JQ9PpbX5m_g

https://www.youtube.com/watch?v=nAdVUXsMQe4

TERM 1, TOPIC 2, LESSON 8

QUADRATIC INEQUALITIES

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners should be able to: ● solve quadratic inequalities.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 7 from the Resource Pack ready.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson have the three examples from point 1 ready.
6. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
11 12	45 51	8 9 10	36 36 37	8 9	35 35	2.16	60	2.8	67

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners should already be familiar with linear inequalities.
2. Linear inequalities will be revised first, introducing the concept of using graphs to understand the answer.
3. This should assist learners in gaining a better understanding of quadratic inequalities.

DIRECT INSTRUCTION

1. Write the following linear inequalities on the board and ask learners to solve them.

1.	$x + 1 < 3$	2.	$2x + 3 > 1$	3.	$-\frac{3}{2}x + 3 \geq 1$

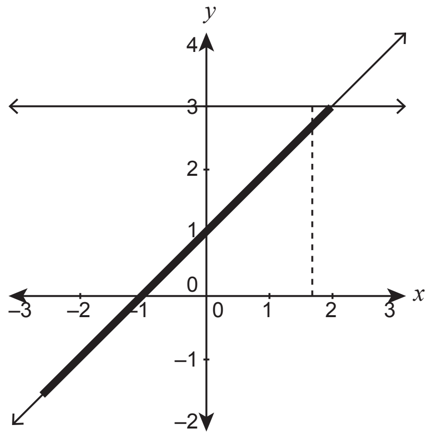
2. Once learners have had a few minutes to solve the inequalities, do the solutions on the board.

1.	$x + 1 < 3$ $x < 2$	2.	$2x + 3 > 1$ $2x > -2$ $x > -1$	3.	$-\frac{3}{2}x + 3 \geq 1$ $-\frac{3}{2}x \geq -2$ $-3x \geq -4$ $x \leq \frac{4}{3}$

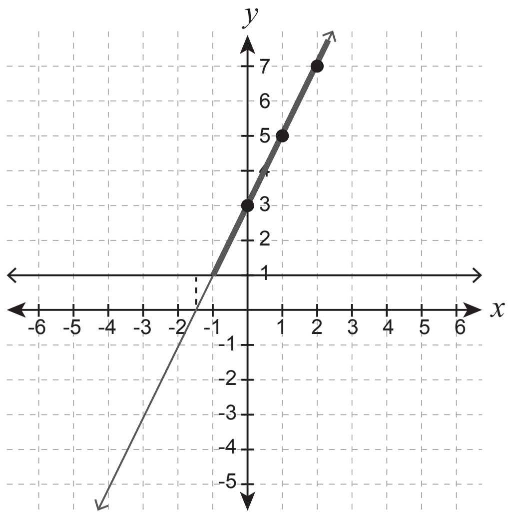
3. Remind learners that although they solve equations algebraically, they also need to remember that, at the same time, they are looking for points of intersection graphically.
4. Say: Let's look at what your solutions from the three examples represent

1.	$x + 1 < 3$	2.	$2x + 3 > 1$	3.	$-\frac{3}{2}x + 3 \geq 1$
What these mean in relation to functions:					
	Where is the line, $y = x + 1$ less than (below) the line $y = 3$		Where is the line, $y = 2x + 3$ greater than (above) the line $y = 1$		Where is the line, $y = -\frac{3}{2}x + 3$ greater than (above) or equal to the line $y = 1$
It is when x is:					
	$x < 2$		$x > -1$		$x \leq \frac{4}{3}$

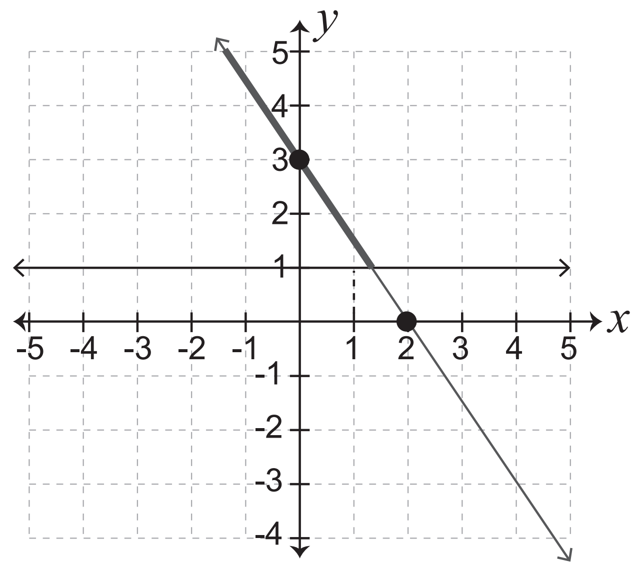
5. Draw the graphical representations of each on the board now and demonstrate what the solution of the inequality means.



Note the two lines represented: $y = x + 1$ and $y = 3$
The bold part of $y = x + 1$ is where this line is underneath (less than) the line $y = 3$.
Moving the focus to what the values of x are for this part of the line, shows that they are all the values less than 2($x < 2$)



Note the two lines represented: $y = 2x + 3$ and $y = 1$
The bold part of $y = 2x + 3$ is where this line is above (greater than) the line $y = 1$.
Moving the focus to what the values of x are for this part of the line, shows that they are all the values greater than -1($x > –1$)



Note the two lines represented: $y = -\frac{3}{2}x + 3$ and $y = 1$.

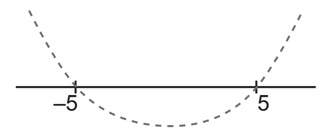
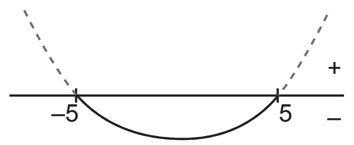
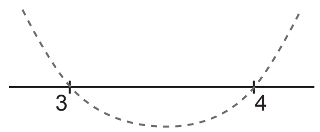
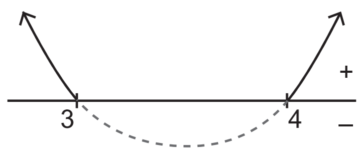
The bold part of $y = -\frac{3}{2}x + 3$ is where this line is above or equal to (greater than or equal to) the line $y = 1$.

Moving the focus to what the values of x are for this part of the line, shows that they are all the values greater than $\frac{4}{3}$ $(x \leq \frac{4}{3})$.

6. Say: *We will consider quadratic inequalities.*

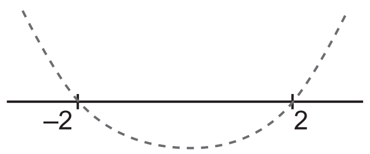
7. Do the following examples in full with learners. They should write them in their books and make notes as they do so. The first example has been done in more detail.

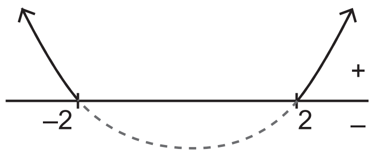
Example	Teaching notes
1. $x^2 < 25$ $x^2 - 25 < 0$ $(x + 5)(x - 5) < 0$ cvs: –5 and 5	Say: *The first step is the same as solving quadratic equations.* Ask: *What needs to be done?* (Get all terms on one side of the sign and factorise). Once this has been completed, tell learners that this is an important step. The values that would have been the solutions if this had been an equation, are now called the CRITICAL VALUES (cvs). They are the values that will assist us in finding the full solution. Ask: *Do you agree that $x^2 - 25$ is a quadratic function that could be represented on the Cartesian plane?* Discuss with learners what the solutions represent from a quadratic function (parabola) being made equal to zero. These would be the x-intercepts.

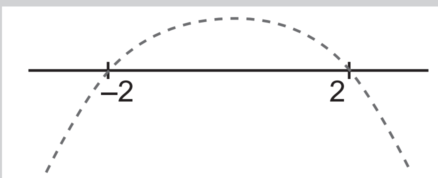
  $-5 < x < 5$	Sketch this function on a number line (which can also be seen as the x-axis). Using the sketch refer learners back to the inequality. $x^2 - 25 < 0$ and ask what it means. For what values of x is the parabola less than zero? In other words, the interest is in where the parabola is negative – below the x-axis. Show where this is by drawing over the dashed curve. Then ask: *What x-values coincide with this part of the quadratic function?* (From –5 to 5). This needs to be represented as an inequality.
2. $x^2 - 7x + 12 > 0$ $(x - 3)(x - 4) > 0$ cvs: 3 and 4   $x < 3$ or $x > 4$	Ask: *What needs to be done first?* (Factorise). Ask: *What can we find now?* (The critical values). Say: *Sketch the quadratic function using the critical values.* Remind learners to look back at the inequality and decide which part of the quadratic function matches the inequality. Ask: *Which part are we interested in?* (Greater than zero). This is the positive part of the function - above the x-axis. Write the inequality.

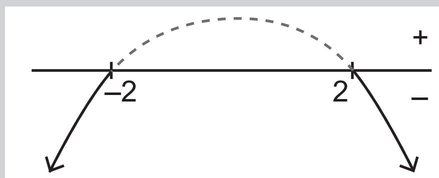
<table>
<tr>
<td>3. $-x^2 + 4 \le 0$
$-1(x^2 - 4) \le 0$

$x^2 - 4 \ge 0$

$(x + 2)(x - 2) \ge 0$
cvs: –2 and 2




$x \le -2$ or $x \ge 2$</td>
<td>Ask: What needs to be done first?
(Factorise).
Say: We need to take out a factor of -1 first.
Ask: What will we need to do next?
(Divide both sides by -1).
Ask: What will we need to remember to do?
(Change the direction of the inequality sign).
Say: Now you can factorise and find the critical values, then draw the sketch and write up the inequality.</td>
</tr>
<tr>
<td colspan="2">Once Example 3 has been completed, discuss it again from another viewpoint.
Point out that learners could have found the critical values and drawn the sketch of the negative quadratic function. If they had done this, they would still have been looking for the values that were less than or equal to zero as the inequality would not have needed to change.
Draw the sketch for learners and show them that the result would have been the same.




The solution would still be x≤-2 or x≥2</td>
</tr>
</table>

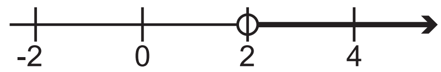
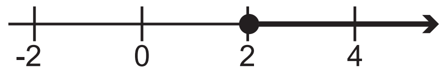
8. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

9. If the learners struggled with writing inequalities, they may need the following summary and/ or examples. These are available in the Resource Pack – Resource 7.

Inequalities, Interval Notation and Representation on a number line

Inequality sign	words	Open/closed dot	
>	Greater than	Open	o⟶
≥	Greater than or equal to	Closed	•⟶
<	Less than	Open	⟵o
≤	Less than or equal to	Closed	⟵•

Examples:

Inequality	Interval notation	
$x > 2$	x∈(2 ; ∞)	
$x \geq 2$	x∈[2 ; ∞)	
$2 \leq x \leq 6$	x∈[2 ; 6]	
$2 < x < 6$	x∈(2 ; 6)	
$2 \leq x < 6$	x∈[2 ; 6)	
$2 < x \leq 6$	x∈(2 ; 6]	

Interval Notation is used to represent a set of Real Numbers as it is impossible to list them.

10. Give learners an exercise to complete on their own.

11. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=XX5oK0fcaDk

https://www.youtube.com/watch?v=8J_m-hMp8lY

TERM 1, TOPIC 2, LESSON 9

NATURE OF ROOTS

Suggested lesson duration: 1,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	30

Lesson Objectives

By the end of the lesson, learners should be able to:

- discuss the nature of roots of a quadratic function.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 8 ready.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson have the quadratic formula ready.
6. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
8	36	13	43	12 13 14 15	40 42 43 45	2.12 2.13	43 45	2.7 2.5	58 51

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Understanding the nature of roots is important in both equations and functions.
2. This is a new concept to learners.

DIRECT INSTRUCTION

1. Write the quadratic formula on the chalkboard:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

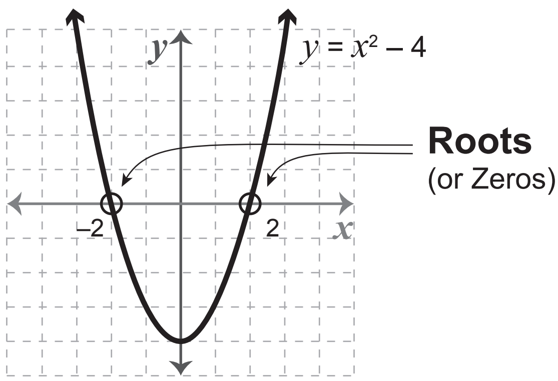
2. Ask: *What does this formula find?*
   (The solutions to a quadratic equation).

3. Point out the heading – Nature of roots. Discuss this with learners and write on the board as you do so.

Nature of roots

When we use the word nature in this context it means the features or character of something

A root is the solution to an equation that is equal to zero, which also means the x-intercepts of a function can be called roots



4. Say: *When we discuss the nature of roots in mathematics, we are describing the types of numbers the roots can be.*

5. Refer learners back to the quadratic formula. Ask: *Which part of this formula is likely to make a difference to the type of answer we could get?*
   (The part underneath the square root sign).

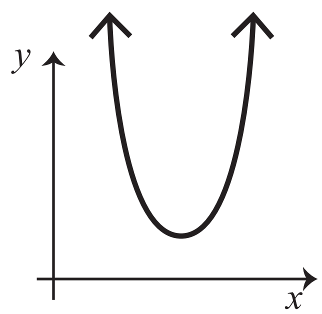
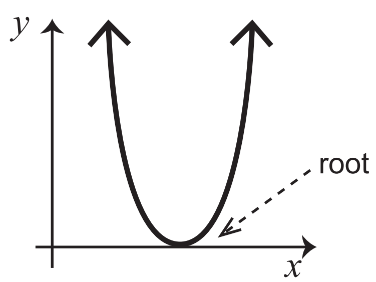
6. Say: *This part* ($b^2 - 4ac$) *is called the discriminant.* Tell learners to write the following in their books:

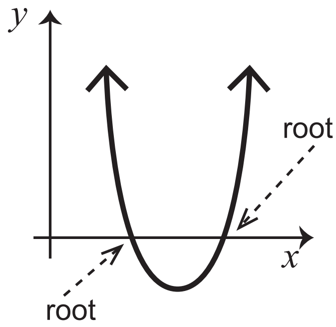
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant

7. Explain that the kind of answer we get inside the square root sign will affect the overall answer from the entire formula.
   Ask: *What type of number would cause a problem under the square root sign?*
   (A negative answer because we cannot find the square root of a negative number).

8. Say: *There are three different possibilities that could occur.*
   Discuss these with learners and tell them to write the information down in their books.

If:	Then:
$b^2 - 4ac < 0$ $x = \frac{-b \pm \sqrt{negative}}{2a}$	There will be no real roots. 
$b^2 - 4ac < 0$ $x = \frac{-b \pm \sqrt{zero}}{2a}$	There will be only one (repeated) root. This can also be described as two equal roots. The root will be real. 

$b^2 - 4ac > 0$ $x = \dfrac{-b \pm \sqrt{positive}}{2a}$	There will be two unequal real roots. 

9. Discuss the final possibility from above when the discriminant is positive. Tell learners that it is possible to give more detail about the nature of the roots in this case.

10. Write the following two possibilities on the board:

$x = \dfrac{-2 \pm \sqrt{25}}{4}$	$x = \dfrac{-2 \pm \sqrt{10}}{4}$

11. Ask: *Which of these would you be happy to calculate without a calculator? Why?* (The first one – there is a perfect square in the square root sign).

12. Tell learners that the first one would give roots that are not only real, but the roots would also be rational. If a function was being drawn, the x-intercepts would also be quite easy to find on the Cartesian plane.

13. Ask learners to find the solutions of the first one now.

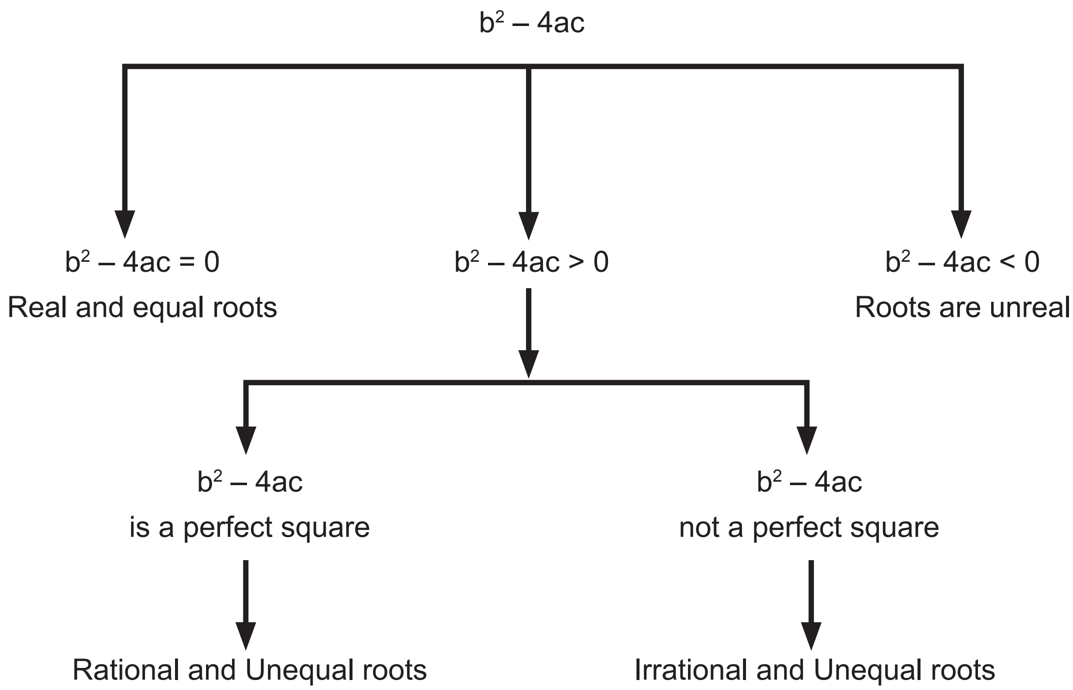
$$\left(x = \frac{3}{4} \quad or \quad x = -\frac{7}{4}\right)$$

14. Point out that the second example has a number that is not a perfect square inside the root sign. It is positive, so the roots will still be real, but they will also be irrational. The answer is currently in its simplified surd form. If a function was being drawn, we would have to find the decimal solution and round it off.

15 Ask learners to find the solutions of the second example and round their answers to two decimal places.

$$(x = 0{,}29 \quad \text{or} \quad x = -1{,}29)$$

16. Before doing a few examples, summarise the information covered using a diagram like the one below. Learners should write the diagram in their books.
The diagram is available in the Resource Pack – Resource 8.



17. Do a few examples with learners.
    Learners should write the examples in their books and make notes as they do so.

Example	Teaching notes
1. Describe the nature of roots for the following equation: $7x^2 - 9x + 2 = 0$ $a = 7 \quad b = -9 \quad c = 2$ $b^2 - 4ac = (-9)^2 - 4(7)(2)$ $= 81 - 56$ $= 25$ The roots are real, rational and unequal	Tell learners that when describing the nature of roots, there is no need to solve the equation. The expression needs to be in standard form so that a, b and c can be stated and $b^2 - 4ac$ can be found. Once the discriminant has been found, learners should note that the answer is positive (the roots are real) and a perfect square (the roots are rational). Remind them there would be two solutions that would also be different (unequal).

2. Describe the nature of roots for the following equation: $25x^2 - 10x + 2 = 0$ $a = 25 \quad b = -10 \quad c = 2$ $b^2 - 4ac = (-10)^2 - 4(25)(2)$ $= 100 - 200$ $= -100$ The roots are non-real	Tell learners to try this one on their own before you complete it with them.
3. Find the value of 'p', if the following quadratic equation has equal roots: $4x^2 - (p - 2)x + 1 = 0$ $a = 4 \quad b = (p - 2) \quad c = 1$ $b^2 - 4ac = 0$ $(p - 2)^2 - 4(4)(1) = 0$ $p^2 - 4p + 4 - 16 = 0$ $p^2 - 4p - 12 = 0$ $(p + 2)(p - 6) = 0$ $p = -2 \quad \text{or} \quad p = 6$	Tell learners that although this question may not look as easy as the previous two, the steps are similar. If roots are mentioned, the discriminant is still of importance. Ask: *Is the expression in standard form?* (Yes). Say: *Write down a, b and c.* Ask: *What should $b^2 - 4ac$ be equal to for an equation to have equal roots?* (Zero). Say: *Make the discriminant equal to zero and solve for p.*

17. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

18. Give learners an exercise to complete on their own.

19. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=5PMzJwkJOjo&list=PLOaNAKtW5HLTSR0eRjIgPOD6HoSBoF-07W&index=6&t=0s

TERM 1, TOPIC 2, LESSON 10

REVISION AND CONSOLIDATION

Suggested lesson duration: 1,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have revised: ● all the concepts covered in Equations and Inequalities.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson have the first few questions ready.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	51 52	Rev	44	Qu’s	46	Rev	68	2.11	81

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Ask learners to recap what they have learned in this section. Spend time pointing out issues that you know are important as well as problems that you encountered from your own learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

This lesson is made up of fully worked examples from a past examination covering most of the concepts in this topic. As you work through these with the learners, it is important to frequently talk about as many concepts as possible.
For example, use the words exponent, surd, prime factors, factorise and rationalise the denominator.

Say: *I am going to do an entire Equations and Inequalities question from the 2015 and 2016 final examination with you. You should write them down as I do them, taking notes at the same time.*

Questions

1. Solve for x in each of the following:
   a) $3x^2 - 5x - 1 = 0$ (leave your answer correct to TWO decimal places)
   b) $x^2 - 6x + 8 = 0$
   c) $4x - 2x^2 < 0$
   d) $2^{3x-1} + 2^{3x} = 12$
   e) $\sqrt{x-1} + 3 = x - 4$
2. Solve for x and y simultaneously:
   $$3x - y + 2 = 0 \text{ and } y = -x^2 + 2x + 8$$
3. Show that the roots of $3x^2 + (k + 2)x = 1 - k$ are real and rational for all values of k.

Teaching notes

1 a) Point out that when the number of decimal places is mentioned, it is usually a clue that the quadratic formula will be used.

b) This should be a simple question for learners. Factorise and solve.

c) Remind learners of the following steps and points to remember:

- Get all terms on one side and factorise (as with quadratic equations)
- State the critical values
- Draw the sketch to represent the quadratic function
- Using the inequality, note the matching part of the function (positive or negative)
- State the solution using the correct inequality.

d) Ask: *How many terms are there on the left-hand side?*

(2).

Say: *We need to factorise. We need to use the approach that was used in simplifying expressions with more than one term. What was the approach?*

(Use the inverse of law 1 to write each base with a single exponent).

Once this has been done, ask: *What do we need to do in the next step?*

(Factorise by taking out a common factor).

Remind learners that bases will need to be equal before making the exponents equal.

e) Remind learners: When solving equations with surds:

- Use inverse operations to get the term with the surd on its own
- Square both sides
- Solve as usual from this step

BUT – remember to ALWAYS check each solution to ensure that both solutions are valid.

2. Remind learners to make one variable the subject of the formula in one of the equations then to use this information to substitute into the other equation. Solve for the unknown variable and substitute back into the other equation to solve for the second variable.

3. Ask: *What would the discriminant need to be for the roots to be real and rational?*

(A perfect square).

Say: *Get the expression on the left-hand side into standard form then state what a, b and c are equal to.*

Once you have a, b and c find $b^2 - 4ac$. Confirm it is a perfect square.

Solutions:

1a) $3x^2 - 5x - 1 = 0$

$a = 3 \quad b = -5 \quad c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{37}}{6}$$

$x = 1{,}85$ or $x = -0{,}18$

b) $x^2 - 6x + 8 = 0$

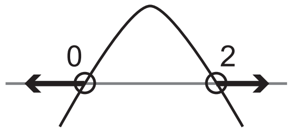
$(x - 4)(x - 2) = 0$

$x = 4$ or $x = 2$

c) $4x - 2x^2 < 0$

$2x(2 - x) < 0$

cvs: 0 ; 2



$\therefore$ $x < 0$ or $x > 2$

d) $2^{3x-1} + 2^{3x} = 12$

$2^{3x}.2^{-1} + 2^{3x} = 12$

$2^{3x}(2^{-1} + 1) = 12$

$2^{3x}\left(\frac{1}{2} + 1\right) = 12$

$2^{3x}\left(\frac{3}{2}\right) = 12$

$2^{3x}\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 12\left(\frac{2}{3}\right)$

$2^{3x} = 8$

$2^{3x} = 2^3$

$\therefore$ $3x = 3$

$x = 1$

e) $\sqrt{x - 1} + 3 = x - 4$

$\sqrt{x - 1} = x - 7$

$(\sqrt{x - 1})^2 = (x - 7)^2$

$x - 1 = x^2 - 14x + 49$

$0 = x^2 - 15x + 50$

$0 = (x - 10)(x - 5)$

$x = 10$ or $x = 5$

n/s

$\therefore$ $x = 10$

2. $3x - y + 2 = 0$ and $y = -x^2 + 2x + 8$

$$3x + 2 = y$$
$$3x + 2 = -x^2 + 2x + 8$$
$$0 = -x^2 - x + 6$$
$$0 = (x + 3)(x - 2)$$
$$x = -3 \quad \text{or} \quad x = 2$$
$$y = 3(-3) + 2 \qquad y = 3(2) + 2$$
$$y = -7 \qquad y = 8$$

3. $3x^2 + (k + 2)x = 1 - k$

$$3x^2 + (k + 2)x + k - 1 = 0$$
$$a = 3 \quad b = (k + 2) \quad c = (k - 1)$$
$$b^2 - 4ac = (k + 2)^2 - 4(3)(k - 1)$$
$$= k^2 + 4k + 4 - 12k + 12$$
$$= k^2 - 8k + 16$$
$$= (k - 4)^2$$

$b^2 - 4ac$ is a perfect square, $\therefore$ the roots are real and rational

Questions

1. Solve for x in each of the following:
   a) $x^2 + x - 12 = 0$
   b) $\sqrt{2x + 1} = x - 1$
   c) $2^{x\sqrt{x}} = 2^{27}$
   d) $x^2 - 2x - 8 < 0$
2. Given $f(x) = 5x^2 + 6x - 7$
   a) Solve for x if $f(x) = 0$ (correct to two decimal places)
   b) Hence, or otherwise, calculate the value of d for which $5x^2 + 6x - d = 0$ has equal roots.
3. Solve for x and y simultaneously
   $x - 2y = -3$ and $xy = 20$

Teaching notes:

1 a) These should be simple questions for learners. Factorise and solve.

b) Remind learners: When solving equations with surds:
- Use inverse operations to get the term with the surd on its own
- Square both sides
- Solve as usual from this step

BUT – remember to ALWAYS check each solution to ensure that both solutions are valid.

c) Say: *This is an exponential equation.*

Ask: *What should you aim for when solving an exponential equation?*

(Get the bases equal so the exponents are equal).

Say: *The bases are already the same, so the exponents can be made equal immediately.*

A knowledge of exponents will then be required.

d) Remind learners of the following steps and points to remember:

Get all terms on one side and factorise (as with quadratic equations)

State the critical values

Draw the sketch to represent the quadratic function

Using the inequality, note the matching part of the function (positive or negative)

State the solution using the correct inequality.

2 a) Learners should recognise that this does not factorise and that they should use the formula.

b) Tell learners that although this question may not look as easy as the previous two, the steps are similar.

If roots are mentioned, the discriminant is still important.

Ask: *is the expression in standard form?*

(Yes).

Say: *Write down a, b and c.*

Ask: *what should $b^2 - 4ac$ be equal to for an equation to have equal roots?*

(Zero).

Say: *Make the discriminant equal to zero and solve for p.*

3. Remind learners to make one variable the subject of the formula in one of the equations then to use this information to substitute into the other equation. Solve for the unknown variable and substitute back into the other equation to solve for the second variable.

Solutions

1a) $x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4$ or $x = 3$

b) $\sqrt{2x+1} = x - 1$

$(\sqrt{2x+1})^2 = (x-1)^2$

$2x + 1 = x^2 - 2x + 1$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0$ or $x = 4$

n/s

$\therefore\ x = 4$

c) $2^{x\sqrt{x}} = 2^{27}$

$\therefore\ x\sqrt{x} = 27$

$x.x^{\frac{1}{2}} = 27$

$x^{\frac{3}{2}} = 27$

$\left(x^{\frac{3}{2}}\right)^2 = 27^2$

$x^3 = (3^3)^2$

$x^3 = 3^6$

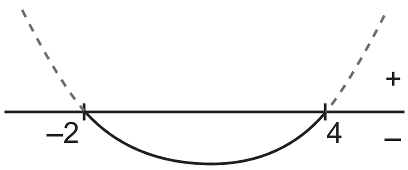
$\sqrt[3]{x^3} = \sqrt[3]{3^6}$

$x = 3^2 = 9$

d) $x^2 - 2x - 8 < 0$

$(x-4)(x+2) < 0$

cv's: 4; –2



$-2 < x < 4$

2a) $0 = 5x^2 + 6x - 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(5)(-7)}}{2(5)}$$

$$x = \frac{-6 \pm \sqrt{176}}{10}$$

$x = 0{,}73$ or $x = -1{,}93$

b) $5x^2 + 6x - d = 0$

$a = 5 \quad b = 6 \quad c = -d$

$b^2 - 4ac = 0$

$(6)^2 - 4(5)(-d) = 0$

$36 + 20d = 0$

$20d = -36$

$d = -\frac{36}{20} = -\frac{9}{5}$

3. $x - 2y = -3$ and $xy = 20$

$x = 2y - 3$

$xy = 20$

$(2y - 3)y = 20$

$2y^2 - 3y = 20$

$2y^2 - 3y - 20 = 0$

$(2y + 5)(y - 4) = 0$

$2y = -5 \qquad y = 4$

$y = -\frac{5}{2}$

$x = 2\left(-\frac{5}{2}\right) - 3 \qquad x = 2(4) - 3$

$x = -8 \qquad x = 5$

1. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.
2. Give learners an exercise to complete on their own.
3. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=wJaiTJ7zSeo&list=PLOaNAKtW5HLTSR0eRjIgPOD6HoSBoF-07W&index=4&t=0s

https://www.youtube.com/watch?v=cAvCCcu3iN0&list=PLOaNAKtW5HLTSR0eRjIgPOD6HoSBoF-07W&index=9&t=0s

Term 1, Topic 3: Topic Overview

NUMBER PATTERNS

A

A. TOPIC OVERVIEW

- This topic is the third of four topics in Term 1.
- This topic runs for two weeks (9 hours).
- It is presented over three lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Number Patterns counts 17% of the final Paper 1 examination.
- Mathematics is especially useful when it helps you predict, and number patterns are all about prediction.
- Working with number patterns leads directly to the concept of functions in mathematics: a formal description of the relationships among different quantities.
- Recognising number patterns is also an important problem-solving skill. If a pattern is recognised when looked at systematically, the pattern can be used to generalise what can be seen in a broader solution to a problem.

Breakdown of topic into 3 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Revision of Linear patterns	2,5	3	Revision and Consolidation	3
2	Quadratic patterns	3,5			

SEQUENTIAL TABLE

B

GRADE 10 and Senior phase	GRADE 11	GRADE 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Linear patterns	• Quadratic patterns	• Arithmetic and Geometric sequences • Arithmetic and geometric series, including sum to infinity • Sigma notation • Derivation of formulae for the sum of arithmetic and geometric series • Problem solving.

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

C

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Number patterns.

These include:

- Attention needs to be paid to the basics of Mathematics – this includes being able to substitute correctly and apply algebraic skills correctly
- The difference between position and value of a term must be emphasised
- Learners need to be exposed to many patterns – including those that include variables
- Learners should be able to establish their own patterns from diagrams or pictures.

It is important that you keep these issues in mind when teaching this section.

While teaching Number Patterns, always use the correct notation and mathematical language. Learners must be encouraged to do the same. A learner's understanding of the concepts is more important than merely doing routine procedures.

D ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test with a memorandum; and an investigation with a rubric are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). These are provided in the Resource Pack – Resources 12 and 13.
- The questions usually take the form of algebraic expressions and fractions that need to simplified or factorised.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

E MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
number pattern	List of numbers that follow a sequence or a pattern
consecutive	One after the other
linear pattern	Pattern formed by adding the same value every time (the value can be positive or negative)
common (or constant) difference	Value added each time to form a linear pattern
quadratic pattern	Sequence of numbers in which the second differences between each consecutive term differ by the same amount
second difference	Second line of differences found in a number pattern. The first line of differences forms a linear pattern and therefore the second line of differences is constant

geometric pattern	Sequence of numbers with a constant ratio between consecutive terms
constant ratio	The number used to multiply one term to get to the next term in a geometric sequence If division occurs, reciprocate and turn it into a multiplication. (Example: $\div 5 = \times \frac{1}{5}$)
rule **n^{th} term** **general term**	Algebraic explanation of how a pattern is formed
term	A number in a given sequence Example: In the sequence: 5 ; 0 ; -5 ; -10 , all four numbers represent terms and each one of them are in a particular position
position	The place in the sequence held by one of the terms Example: In the sequence: 2 ; 4 ; 6 ; 8…. 6 is in the 3^{rd} position

TERM 1, TOPIC 3, LESSON 1

REVISION OF LINEAR NUMBER PATTERNS

Suggested lesson duration: 2,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have revised: • linear number patterns.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	54	1	47	Qu’s	49	3.1	71	3.1	89

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. A generous amount of time is allocated to this topic. Use the time wisely by allowing learners to practice concepts from previous years until they are fully confident in them.
2. There is time to show learners why we use the formulae that we do to find the general term. Understanding where a formula comes from always contributes to a deeper conceptual understanding.

DIRECT INSTRUCTION

1. Start the lesson by asking: *What is a linear number pattern?*
   (A pattern that has a common difference between each term)
2. Ask for a volunteer to come and write an example of a linear number pattern on the board. Discuss the pattern with learners, ask: *What is the common difference? What would the next three terms be?*
3. Remind learners that, in general, there are three types of basic questions related to number patterns. These are: finding the general term (often called the n^{th} term); finding the position of a term given; finding a term in a given position.

   > Note:
   > Discuss 'position' with learners. Point out that a position can only ever be positive and a natural number. It is not possible to have a term in a negative position or in a fractional position.
   > The diagnostic reports state that learners often confuse the term and the position.
   > Spend some time on the concepts 'term' and 'position' and ask directed questions using any patterns covered throughout the rest of the lesson on the board to ensure that learners understand the difference.
   > For example, ask,
   > *In this pattern, what term is in the 3^{rd} position?*
   > *In this pattern, what is the position of the term …?*

4. Do an example covering all three of these types of questions as well as two others. Learners should write the example in their books, making notes as they do so.

Example:

Use the linear pattern, 18; 15; 12… to answer the following questions:

a) List the next three terms
b) Find the n^{th} term
c) Find the 25^{th} term in the sequence
d) In which position would –15 lie in the sequence?
e) Is –52 part of the sequence?

Teaching notes:

a) Remind learners that a question like this would just require counting.
b) Use the method taught in Grade 10.
   Remind learners that they need to find the common difference (b). then use Term 1 (T_1) to find c.
   Show learners the alternative method when the example is complete.
c) Ask: *What algebraic skill will be used to find a certain term in the sequence if given position?*
   (Substitution).
d) Ask: *What algebraic skill will be used to find the position of a given term?*
   (Solving equations)
e) Ask learners for ideas how to answer this question. Listen to their responses – they may have a perfectly sound way to find the solution. Listen carefully to establish whether learners have the conceptual knowledge.
   After the discussion, ask: *What types of numbers can n possibly be?*
   (Only natural numbers – position cannot be a negative number or a fraction).
   Say: *This is how we will approach this question. We will solve for n. If the answer is a natural number, then –52 is part of the sequence. If n is not a natural number, then –52 is not part of the sequence.*

Solutions:

a) 9 ; 6 ; 3
b) Common difference: –3

$T_n = bn + c$

$\therefore\ T_n = -3n+c$

$T_1 = -3(1)+c$

$18 = -3+c$

$21 = c$

$\therefore\ T_n = -3n+21$

c) $T_n = -3n+21$

$T_{25} = -3(25)+21$

$T_{25} = -75+21$

$T_{25} = -54$ $\quad\therefore$ the 25^{th} term of the sequence is –54

d) $T_n = -3n+21$

$-15 = -3n+21$

$3n = 21+15$

$3n = 36$

$n = 12$ $\therefore$ -15 is in the 12^{th} position

e) $T_n = -3n+21$

$-52 = -3n + 21$

$3n = 21 + 52$

$3n = 73$

$n = \frac{73}{3}$ $\therefore$ -52 is not in the sequence because $\frac{73}{3}$ is not a natural number

5. Show learners where the general term, $T_1 = bn + c$ comes from.

6. Write $T_1 = bn + c$ on the board. Say: *Substitute* $n = 1$ *to find the first term.* When this has been done, say: *Find the 2^{nd}, 3^{rd} and 4^{th} terms by substituting* $n = 2$; $= 3$; $n = 4$.

7. Learners should have the following terms: $b + c$; $2b + c$; $3b + c$ and $4b + c$
Write these on the board, doing each substitution if necessary.

8. Work through finding the difference between each term with learners.

(2b+c-(b+c) ; 3b+c-(2b+c) ; 4b+c-(3b+c)

b + c 2b + c 3b + c 4b + c

b b b

9. Say: *We can therefore draw the following conclusions:*

$T_1 = b + c$

The constant difference is b.

10. Using the linear pattern dealt with in the example, show learners how this helped us find the general term:

18 ;15 ;12…

$T_1 = b + c$
$18 = b + c$
But b is the constant difference which is -3
$\therefore$ $18= -3 + c$
$\therefore$ $21 = c$

11. Show learners another way to find the general term of a linear sequence.

12. Write the following sequence on the board: 7 ;12 ;17 ;22…

13. Draw up the following table:

Term	Made up of:	Position of term	How to find each term	Teaching notes
7	7	1	7 + 0	
12	7 + 5	2	7 + 1(5)	The first term plus one of the constant difference
17	7 + 5 + 5	3	7 + 2(5)	The first term plus two of the constant difference
22	7 + 5 + 5 + 5	4	7 + 3(5)	The first term plus two of the constant difference
Note: To find each term, we need to add a number of constant differences. The number of constant differences is ONE LESS than the position required. In other words, to find the 4th term, we need to add THREE constant differences.				

14. Show learners that we must then have the following general term:

$$T_n = 7 + (n - 1)(5)$$

Label what has been found:

$$T_n = 7 + (n - 1)(5)$$

First term (T_1) Constant difference

15. Change what has been found into a general formula:

$$T_n = a + (n - 1)d$$

where a is the first term and d is the common difference.

16. Tell learners that this formula is available on the formula sheet in Grade 12.

17. Show learners another type of example in which some of the terms are unknown

Example:

Consider the linear pattern: 14 $\quad x \quad$ 24 $\quad y$

Find x and y

Teaching notes:

Point out that the key information is that this is a linear pattern.

Ask: *What do you know about a linear pattern?*

(There is a constant difference between the consecutive terms).

Find the differences in terms of x and y with the learners.

Once this has been done, ask again: *What do we know about each of these expressions?*

(they are equal to each other).

Say: *We can use this to form an equation and solve for x. Once we have the value of x, we can use it to form another equation and solve for y.*

Once the solution has been completed, tell learners they should check their answer by substituting the values found and confirm that it is a linear sequence.

Solution:

$$14 \qquad x \qquad 24 \qquad y$$

$$x - 14 \qquad 24 - x \qquad y - 24$$

$$\begin{aligned} x - 14 &= 24 - x \\ 2x &= 38 \\ x &= 19 \end{aligned} \qquad\qquad \begin{aligned} y - 24 &= 24 - x \\ y - 24 &= 24 - 19 \\ y &= 29 \end{aligned}$$

18. Do one more example using a question from a previous Grade 10 examination. Learners should write the example in their books, making notes as they do so.

1. Given the linear number pattern: 8 ;3 ; –2…
   a) Write down the NEXT TWO terms of the pattern
   b) Determine the n^{th} term of the pattern
   c) Calculate T_{30}, the thirtieth term of the pattern
   d) Which term of the pattern is equal to –492
2. The first four terms of PATTERN A and PATTERN B are shown in the table below:

Position of term (n)	1	2	3	4
PATTERN A	1	3	5	7
PATTERN B	1	9	25	49

   a) Determine a formula for the n^{th} term of PATTERN A
   b) Hence, or otherwise, determine a general formula for the 6^{th} term of PATTERN B
   c) Hence, determine a general formula for the pattern 0 ; –6 ; –20 ; –42…
      Simplify your answer as far as possible.

Teaching notes:

1. a) Learners should find this very straightforward.
   b) Learners can try any of the two methods you have done with them.
   Suggest that they try both methods and confirm that they get the same answer.
   c) Ask: *How do we find a term in a given position?*
   (Substitute the position given for n).
   d) Ask: *How do we find the position of a given term?*
   (Substitute the term for T_n and solve for n).
2. a) This is similar to question 1b – suggest again to learners that they use both methods.
   b) Say: *Note the word 'hence' – this implies that the answer to the previous question should be of assistance.*
   Ask: *What connection is there between Pattern A's terms and Pattern B's terms?*
   (Pattern B's terms are the squares of Pattern A's terms)
   Say: *In this case we can just square the general term that we found for Pattern A.*
   c) Say: *Notice the 'hence' again – the solution to the previous question will be required.*
   Suggest that learners fill in the new pattern of numbers in a new row at the bottom of the table.
   Ask: *What is the connection between the new pattern and the patterns given?*
   (The new pattern is the difference between the other two patterns).

Solutions:

1. a) $-7\ ;\ -12$
   b) $T_n = -5n + 13$
   c) $T_n = -5n + 13$
   $T_{30} = -5(30) + 13$
   $T_{30} = -137$
   d) $T_n = -5n + 13$
   $-492 = -5n + 13$
   $5n = 13 + 492$
   $5n = 505$
   $n = 101$
2. a) $T_n = 2n - 1$
   b) $T_n = (2n - 1)^2$
   $T_n = 4n^2 - 4n + 1$
   c) $T_n = (2n - 1) - (2n - 1)^2$
   $T_n = 2n - 1 - (4n^2 - 4n + 1)$
   $T_n = 2n - 1 - 4n^2 + 4n - 1$
   $T_n = -4n^2 + 6n - 2$

19. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

20. Give learners an exercise to complete on their own.

21. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=UuceRRQGk8E

(Linear sequences – n^{th} term)

TERM 1, TOPIC 3, LESSON 2

QUADRATIC PATTERNS

Suggested lesson duration: 3,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	30

Lesson Objectives

By the end of the lesson, learners should be able to:

- recognise a quadratic pattern
- find the general term of a quadratic pattern
- find a term in a given position of a quadratic pattern
- find the position of a term in a quadratic pattern.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the first quadratic pattern (point 2).
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	58	2 3	48 49	1	56	3.2 3.3	72 75	3.2 3.3	91 98

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. This is a new concept for learners. There is plenty of time to go through the concept slowly and methodically, ensuring that each learner understands each new part.
2. Once the lesson is complete and learners have completed the exercises in their textbook, give them a second exercise to do (from a different textbook or a past test paper) – this will give them many opportunities to do as many types of questions as possible.

DIRECT INSTRUCTION

1. Say: *Today we are going to encounter a new type of number pattern.*
2. Write the following pattern on the board:

   2 ;7 ;16 ;29

3. Ask: *Why is this NOT a linear pattern?*
   (It doesn't have a common difference).
4. Tell learners to find the difference between all the terms anyway.

   5 ;9 ;13

5. Ask: *What do you notice about this new set of numbers?*
   Do they form a pattern that you recognise?
   (The differences form a linear pattern).
   Ask: *What is the constant difference?*
   (4).
6. Say: *Let's show these two sets of differences:*

   2 7 16 29
   5 9 13
   4 4

   Say: *We call this a second constant difference.*
   We will come back to this shortly.

7. Write the following general term of a number pattern on the board:

$$T_n = 3n^2 + 3n + 1$$

Ask learners to find the first 4 terms (T_1, T_2, T_3, T_4)
After a few minutes show the full solutions on the board

$T_n = 3n^2 + 3n + 1$	$T_n = 3n^2 + 3n + 1$	$T_n = 3n^2 + 3n + 1$	$T_n = 3n^2 + 3n + 1$
$T_1 = 3(1)^2 + 3(1) + 1$	$T_2 = 3(2)^2 + 3(2) + 1$	$T_3 = 3(3)^2 + 3(3) + 1$	$T_4 = 3(4)^2 + 3(4) + 1$
$T_1 = 1$	$T_2 = 7$	$T_3 = 19$	$T_4 = 37$

8. Ask learners to find the first and second line of differences as was done in the example above.

$$\begin{array}{ccccccc} 1 & & 7 & & 19 & & 37 \\ & 6 & & 12 & & 18 & \\ & & 6 & & 6 & & \end{array}$$

Point out that this number pattern has a second line of difference – this makes it a quadratic number pattern

9. Say: *From this we can reason that the general term of a quadratic number pattern is in the form:*

$$T_n = an^2 + bn + c$$

10. Say: *Let's look at how we will find a, b and c.*
Tell learners to find T_1, T_2, T_3 and T_4 of the general term. Remind them that their answers will be in terms of a and b.

11. After a few minutes, show the full solutions on the board.

$T_n = an^2 + bn + c$ $T_1 = a(1)^2 + b(1) + c$ $T_1 = a + b + c$	$T_n = an^2 + bn + c$ $T_2 = a(2)^2 + b(2) + c$ $T_2 = 4a + 2b + c$
$T_n = an^2 + bn + c$ $T_3 = a(3)^2 + b(3) + c$ $T_3 = 9a + 3b + c$	$T_n = an^2 + bn + c$ $T_4 = a(4)^2 + b(4) + c$ $T_4 = 16a + 4b + c$

12. Write these terms on the board in a line ready to find the first and second line of differences. Ask learners to take it down in their books.

> Ask learners to work out the differences in their books. Do the full calculations on another part of the board if necessary (if some learners are getting the incorrect answers)

$$\begin{array}{ccccccc}
a+b+c & & 4a+2b+c & & 9a+3b+c & & 16a+4b+c \\
 & 3a+b & & 5a+b & & 7a+b & \\
 & & 2a & & 2a & &
\end{array}$$

13. Ask learners to note (and write in their books) that:

$a + b + c =$	T_1
$3a + b =$	the first difference in the first line of difference (the difference between T_2 and T_1)
$2a =$	second constant difference

14. This information is going to be used to help us find the general term of a quadratic number pattern.

15. Ask learners to refer back to the quadratic equation that we started with:

$$\begin{array}{ccccccc}
2 & & 7 & & 16 & & 29 \\
 & 5 & & 9 & & 13 & \\
 & & 4 & & 4 & &
\end{array}$$

Say: *Now we are going to find the general term for this pattern.*

Point out that we need to start by finding a then b then c. Once a has been found, it will be used to find b. Once b has been found, both a and b will be used to find c.

16. Do the full working with learners now, explaining as each step is done.

$2a = 4$ $\therefore\ a = 2$	$3a + b = 5$ $3(2) + b = 5$ $b = -1$	$a + b + c = 2$ $2 - 1 + c = 2$ $c = 1$

$\therefore$ the general term of the above quadratic number pattern is: $T_n = 2n^2 - 1n + 1$
In simplest form: $T_n = 2n^2 - n + 1$

17. Do another example with learners:
Example: Find the n^th term of the following number pattern: 7 ; 10 ; 15 ; 22

7 10 15 22
3 5 7
2 2

$2a = 2$ $\therefore\ a = 1$	$3a + b = 3$ $3(1) + b = 3$ $b = 0$	$a + b + c = 7$ $1 + 0 + c = 7$ $c = 6$

$\therefore$ the general term of the above quadratic number pattern is: $T_n = 1n^2 + 0n + 6$
In simplest form: $T_n = n^2 + 6$

18. Give learners these examples to do on their own:

4 7 12 19 28	2 8 18 32 50

Solutions:

4 7 12 19 28 3 5 7 9 2 2 2	2 8 18 32 50 6 10 14 18 4 4 4
$T_n = 1n^2 + 0n + 3$ $T_n = n^2 + 3$	$T_n = 2n^2 + 0n + 0$ $T_n = 2n^2$

19. Do the solutions in full on the board to ensure that learners have four complete examples in their books.

20. Use some of these general terms to answer other questions.

Example: Consider the general term $T_n = n^2 + 3$ a) Find T_6 and T_{10} b) In which position does the term 147 lie?
Teaching notes: Ask: *What algebraic skill is required to find the value of a term when given the position?* (Substitution). Ask: *What algebraic skill is required to find the position of a term given?* (Solving an equation). When working through the solution of b): Once the equation has been set up, ask: *What type of equation is this?* (A quadratic equation). Ask: *What does this mean?* (That there should be two solutions). Ask: *Can a term be in two different positions?* (No). Say: *Let's solve the equation and discuss the solutions.*
Solution: a) $T_6 = (6)^2 + 3$ $T_6 = 39$ $T_{10} = (10)^2 + 3$ $T_{10} = 103$ b) $T_n = n^2 + 3$ $147 = n^2 + 3$ $0 = n^2 - 144$ $0 = (n + 12)(n - 12)$ $n = -12$ or $n = 12$ $\therefore$ 147 is in the 12^{th} position (position cannot be negative).

21. Ask learners to try the following question on their own:

Example:
Consider the general term $T_n = 2n^2 - 1n + 1$
a) Find T_5 and T_9
b) In which position does the term 232 lie?

Solutions:

Solution:
a) $T_5 = 2(5)^2 - 5 + 1 = 46$
$T_9 = 2(9)^2 - 9 + 1 = 154$
b) $T_n = 2n^2 - n + 1$
$232 = 2n^2 - n + 1$
$0 = 2n^2 - n - 231$
$0 = (2n + 21)(n - 11)$
$n = -\frac{21}{2}$ or $n = 11$
∴ 232 is in the 11th position (position cannot be negative or a fraction).

22. Do one final example before learners try an exercise on their own.

Example:
2 is the constant second difference of a quadratic number pattern:
5 x 17 y 37
Find x and y

Teaching notes:
Ask: *What can we do to find the values required?*
(Find the first differences in terms of x and y then use the differences between these to equal the second constant difference given and form equations).
Once the solution has been completed, tell learners they should check their answer by substituting the values found and confirm that it is a quadratic sequence.

Solution:

$$
\begin{array}{ccccccccc}
5 & & x & & 17 & & y & & 37 \\
& \diagdown\ \diagup & & \diagdown\ \diagup & & \diagdown\ \diagup & & \diagdown\ \diagup & \\
& x-5 & & 17-x & & y-17 & & 37-y & \\
& & \diagdown\ \diagup & & \diagdown\ \diagup & & \diagdown\ \diagup & & \\
& & 2 & & 2 & & 2 & &
\end{array}
$$

$\therefore$ $17 - x - (x - 5) = 2$ and $37 - y - (y - 17) = 2$

$$17 - x - x + 5 = 2 \qquad\qquad 37 - y - y + 17 = 2$$
$$-2x = -20 \qquad\qquad -2y = -52$$
$$x = 10 \qquad\qquad y = 26$$

23. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

24. Give learners an exercise to complete on their own.

25. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=FfCq7bGAFoY

(Quadratic sequences – n^{th} term)

TERM 1, TOPIC 3, LESSON 3

REVISION AND CONSOLIDATION

Suggested lesson duration: 3 hours

A POLICY AND OUTCOMES

CAPS Page Number	30
Lesson Objectives By the end of the lesson, learners will have revised: • all the patterns covered in Grade 10 and Grade 11.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the question for the first worked example.
5. The table below provides references to this topic in Grade 11 textbooks.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	59 61	Rev	54	Qu’s	58	Rev	78	3.4	99

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Although there has only been one new concept covered in number patterns, time spent revising is never wasted. If time permits, when learners have completed the revision exercise source a past test or another past examination question for them to do.

DIRECT INSTRUCTION

1. Tell learners that before they do a revision exercise on number patterns, you are going to work through a past examination paper question with them.

2. Learners should write it in their books.

Consider the quadratic pattern: -9 ; -6 ;1 ;12 ;x ;… a) Determine the value of x b) Determine a formula for the n^{th} term of the pattern. c) A new pattern, P_n, is formed by adding 3 to each term in the given quadratic pattern. Write down the general term of P_n in the form $P_n = an^2 + bn + c$. d) Which term of the sequence found in c) has a value of 400?
Teaching notes: a) Ask: *What will we need to do to find the value of x?* (Find the first and second constant difference. The second differences should be the same). b) Ask: *How can we find the general term?* (Use $2a = 2^{\text{nd}}$ difference; $3a + b$ = first difference; $a + b + c = 1^{\text{st}}$ term) c) Say: *Think about a quadratic function (the parabola).* Ask: *What happens if each point (term) increases by 3?* (The function shifts up 3 units). Ask: *How would we show that on the function?* (Add 3 to the entire function – at the end). d) Ask: *What algebraic skill is required to find the position of a given term?* (Solving equations - substitute the term for T_n and solve for n) Ask: *A quadratic equation will give two solutions – how will we know which one is correct?* (A position can only be a natural number).

Solutions:

a)

$$\begin{array}{ccccccccc} -9 & & 6 & & 1 & & 12 & & x \\ & 3 & & 7 & & 11 & & x-12 & \\ & & 4 & & 4 & & x-23 & & \end{array}$$

$x - 23 = 4$

$x = 27$

b)

$2a = 4$ $a = 2$	$3a + b = 3$ $3(2) + b = 3$ $b = -3$	$a + b + c = -9$ $2 - 3 + c = -9$ $c = -8$

$\therefore\ T_n = 2n^2 - 3n - 8$

c) $T_n = 2n^2 - 3n - 8$

$\therefore\ P_n = 2n^2 - 3n - 8 + 3$

$P_n = 2n^2 - 3n - 5$

d) $P_n = 2n^2 - 3n - 5$

$400 = 2n^2 - 3n - 5$

$0 = 2n^2 - 3n - 5 - 400$

$0 = 2n^2 - 3n - 405$

$0 = (2n + 27)(n - 15)$

$n = -\frac{27}{2}$ or $n = 15$

$\therefore$ 400 is in the 15th position.

1. Given the linear pattern: 18 ;14 ;10 ;…
   a) Write down the fourth term.
   b) Determine the formula for the general term of the pattern.
   c) Which term of the pattern will have a value of –70?
   d) If this linear pattern forms the first differences of a quadratic pattern, Q_n, determine the first difference between Q_{509} and Q_{510}.
2. A quadratic pattern has a constant second difference of 2 and $T_5 = T_{17} = 29$
   a) Does this pattern have a minimum or a maximum value? Justify your answer.
   b) Determine an expression for the nth term in the form $T_n = an^2 + bn + c$

Teaching notes:

1.a) Ask: *What do we need to calculate to find the next term in the pattern?*
(Find the common difference then add it to the last term given – that is, the third term).

b) Ask: *What do we need to find the general term?*
(The first term and the common difference).
Remind learners that there are two methods of finding the general term

c) Ask: *What algebraic skill is required to find the position of a given term?*
(Solving equations - substitute the term for T_n and solve for n).

d) Learners may find this a challenge but with a careful explanation they should realise quickly that it is quite simple. Write a quadratic pattern on the board (the square numbers: 1 ;4 ;9 ;16 ..).
Tell learners this pattern will be called Q_n.
Ask learners for the first differences (3 ;5 ;7 ;…).
Tell learners this pattern will be called T_n.
Point out that our focus is the linear pattern formed but we also need to see which terms make up each difference.
Ask: *Which terms make the 1st term in the new linear pattern?*
(Q_1 and Q_2).
Which terms make the 2nd term in the new linear pattern?
(Q_2 and Q_3).
Explain, or make the rule that can assist us in seeing the connection between terms in the quadratic pattern and the terms in the linear pattern.
($Q_n - Q_{n-1} = T_{n-1}$)
What term in the linear sequence are we interested in to answer the question?
(T_{509})

2. a) Ask: *How do we know if a quadratic function has a minimum or maximum value?*
(If $a < 0$, then there is a maximum value and if $a > 0$, then there is a minimum value).
Say: *we need to find the value of a in the quadratic pattern to answer this question.*

b) Ask: *What information do we have about the quadratic pattern?*
(The 5th and 17th terms are both equal to 29).
Ask: *What else do we already know?*
($a = 1$).
Say: *Write two statements using the general formula for the quadratic number pattern using* $T_5 = 29$ *and* $T_{17} = 29$. *Then use simultaneous equations to find* b *and* c.

Solutions:

1.a)

$$\begin{array}{ccccc} 18 & & 14 & & 10 \\ & -4 & & -4 & \end{array}$$

$T_4 = 6$

b) $T_n = a + (n - 1)d$

$T_n = 18 + (n - 1)(-4)$

$T_n = 18 - 4n + 4$

$T_n = -4n + 22$

c) $T_n = -4n + 22$

$-70 = -4n + 22$

$-70 - 22 = -4n$

$-92 = -4n$

$n = 23$

$\therefore$ –70 is in the 23rd position.

d) $Q_{510} - Q_{509} = T_{509}$

$T_{509} = -4(509) + 22$

$T_{509} = -2014$

2.a) $2a = 2$

$a = 1$

$a > 0 \ \therefore$ this pattern has a minimum value

b) $T_n = an^2 + bn + c$

$T_5 = 1(5)^2 + b(5) + c$	$T_{17} = 1(17)^2 + b(17) + c$
$T_5 = 25 + 5b + c$	$T_{17} = 289 + 17b + c$
$29 = 25 + 5b + c$	$29 = 289 + 17b + c$
$4 = 5b + c$	$-260 = 17b + c$

$c = 4 - 5b$

$\therefore -260 = 17b + 4 - 5b$

$-260 - 4 = 17b - 5b$

$-264 = 12b$

$b = -22$

$\therefore c = 4 - 5(-22)$

$c = 114$

$\therefore T_n = n^2 - 22n + 114$

3. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

4. Give learners an exercise to complete with a partner.

5. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 1, Topic 4: Topic Overview

ANALYTICAL GEOMETRY

A. TOPIC OVERVIEW

- This topic is the fourth of four topics in Term 1.
- This topic runs for three weeks (13,5 hours).
- It is presented over five lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 12,5 hours). For example, one lesson in this topic could take three school lessons. Plan according to your school's timetable.
- The 1 hour not accounted for can be used for the Term 1 test.
- Analytical Geometry counts 20% of the final Paper 2 examination.
- It is important that you constantly remind learners of the connection between analytical geometry, functions and equations.
- Algebraic manipulation (for example, substitution) and an understanding of linear functions are important skills for this section..

Breakdown of topic into 4 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Revision	3	4	Angle of inclination	3
2	Equations of straight lines	2,5	5	Revision and Consolidation	3
3	Investigation	1		Term 1 Test	1

SEQUENTIAL TABLE

B

GRADE 10 and Senior phase	GRADE 11	GRADE 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Plotting point on a Cartesian plane • Distance formula • Gradient of a line segment • Mid-point of a line segment	Derive and apply the: • equation of a line through two points • equation of a line through one point and parallel or perpendicular to another • inclination of a line	Derive and apply the: • equation of a circle • equation of a tangent to a circle at a given point

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

C

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Analytical Geometry.

These include:

- basic errors with signs and computation
- copying formulae from the information sheet incorrectly
- lack of knowledge of Euclidean Geometry in general (needed to answer Analytical Geometry questions)
- not giving reasons for statements
- confusing perpendicular lines with parallel lines.

It is important that you keep these issues in mind when teaching this section.

While teaching Analytical Geometry, remind learners that a knowledge of other aspects of the curriculum is important. For example, knowing the properties of quadrilaterals from Euclidean Geometry and finding the equation of a straight line from Functions are both required in this section.

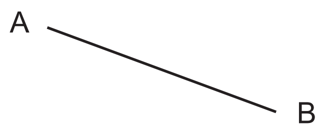
D ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test with a memorandum; and an investigation with a rubric are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). These assessments are available in the Resource Pack – Resources 12 and 13.
- The questions usually take the form of algebraic expressions and fractions that need to simplified or factorised.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

E MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	**Explanation**
distance	Length (in units) from one point to another Found by using the distance formula using two points given
gradient	How steep a line is Found by using the gradient formula using two points given
mid-point	The co-ordinate that represents the middle of a line segment Found by using the mid-point formula using two points given
parallel	Lines that have the same gradient are parallel to each other
perpendicular	At a right angle
x-intercept	Point at which a graph cuts the x-axis
y-intercept	Point at which a graph cuts the y-axis
point of intersection	The co-ordinate where two graphs intersect each other

diagonal	Line segment joining opposite corners of a quadrilateral
rectangle	4-sided shape (quadrilateral) where both pairs of opposite sides are equal in length and all 4 angles are 90º
square	4-sided shape (quadrilateral) where all 4 sides are equal in length and all 4 angles are 90º
kite	4-sided shape (quadrilateral) where the adjacent sides (sides next to each other) are equal in length The diagonals are perpendicular to each other
rhombus	Parallelogram with 4 equal sides
parallelogram	4-sided shape (quadrilateral) that has two pairs of parallel sides
equilateral triangle	Triangle with 3 equal sides and 3 equal angles
isosceles triangle	Triangle with 2 equal sides and 2 equal angles
collinear	Points that lie on the same line
origin	Point where the x and y axis meet on a Cartesian plane
line segment	All points between two given points 
perimeter	Distance around the outside of a shape Length of the outline of the shape
equidistant	Exactly the same distance
angle of inclination	Angle between a line and the horizontal line (most often the x-axis) Can be any measurement from 0º to 180º Always measured from the horizontal line in an anti-clockwise direction If the line has a positive gradient, the angle of inclination will be less than 90º If the line has a negative gradient, the angle of inclination will be between 90º and 180º

TERM 1, TOPIC 4, LESSON 1

ANALYTICAL GEOMETRY - REVISION

Suggested lesson duration: 3 hours

A POLICY AND OUTCOMES

CAPS Page Number	31
Lesson Objectives By the end of the lesson, learners will have revised finding the: • distance between two points • gradient of a line segment • mid-point of a line segment.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw a Cartesian plane.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev 1	64 65			Qu's	60	4.1	84	4.1	112

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. This is a topic that learners can do well in. Learners are given all formulae and the time allocated is generous.
2. Tell learners that there is plenty of time allocated to this topic so they will have time to revise work covered in Grade 10 and investigate new ideas to gain a good understanding of the concepts covered in Grade 11.

DIRECT INSTRUCTION

1. Ask: *What are the three concepts you learned in Grade 10 Analytical Geometry?* (Distance, midpoint and gradient of a line segment). Praise any learners who remembered correctly.
2. Ask learners to draw a Cartesian plane in their books. Plot the following points on the Cartesian plane and ask learners to do the same.

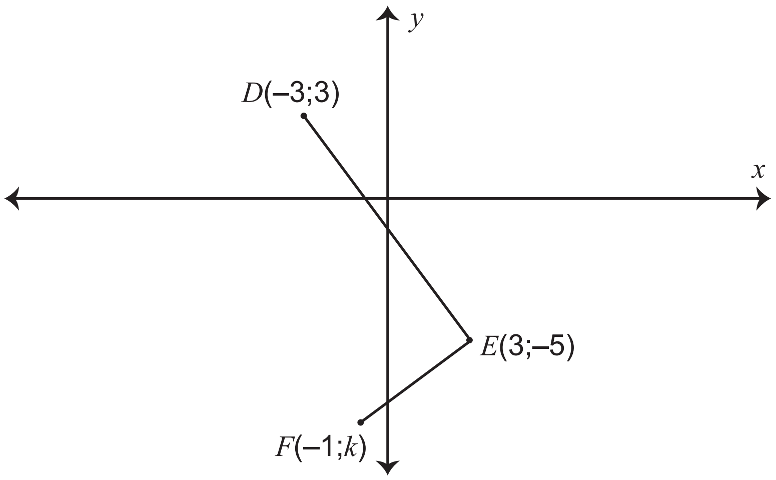
   A (-4;-1) B (-2;2) C (5;-1) D (-4;5)

3. Ask learners to answer the following questions. Tell them to try it on their own as you would like them to establish for themselves how much they remember from Grade 10.
   1) Find the length of line segment AB
   2) Find the gradient of AB and CD
   3) Find the midpoint of CD
   4) What does your answer to (2) tell you about the lines AB and CD?
4. Walk around and assist learners where necessary.
5. After 10 – 15 minutes, give learners the following solutions and ask if anyone would like you to demonstrate on the board.
   Solutions:
   1) AB = $\sqrt{37}$ = 6,08 units
   2) $mAB = \frac{3}{2}$ and $mCD = \frac{-2}{3}$
   3) Midpoint CD: $\left(\frac{1}{2};2\right)$
   4) $AB \perp CD$ (because $\frac{3}{2} \times \frac{-2}{3} = -1$)
6. If any learners need the questions done in full on the board, do so now.

7. Tell learners that, in order to ensure they are completely confident in all the skills they require before they move on to Grade 11 work, you are going to do two fully worked examples with them from previous Grade 10 examinations. Tell learners to write them in their books and work through them with you. Suggest that learners make notes, adding anything they think they may need to refer to at a later stage.

Example 1

In the diagram below, $D(-3\ ;\ 3)$, $E(3\ ;\ -5)$ and $F(-1\ ;\ k)$ are three points in a Cartesian plane.



a) Calculate the length of DE
b) Calculate the gradient of DE
c) Determine the value of k, if $D\hat{E}F = 90°$
d) If $k = -8$, determine the co-ordinates of M, the mid-point of DF.
e) Determine the co-ordinates of a point G such that the quadrilateral DEFG is a rectangle.

Teaching notes:

a) and b)

Learners should be able to do these questions without assistance using the distance and gradient formulae.

c)

Ask: *What aspect of Analytical Geometry is required to answer this question?* (Gradient).

Ask: *What do we know about the gradients of two lines that form a 90°?*
(The product of their gradients is equal to –1).

d)

Learners should be able to do these questions without assistance using the midpoint formula.

e)

Remind learners that the order given for the rectangle is important.

Ask: *Look at the sketch. Where do you expect the coordinate to be?*

(Near the x-axis in the 2nd or 3rd quadrant or possibly even on the x-axis).

Ask: *What aspect of Analytical Geometry is required to answer this question?*

(Diagonals bisect each other so the midpoint found in the previous question can be used to find the point G. It will need to be used in reverse because M is the midpoint of EG).

Say: *The point could also be found by inspection (using the gradients of opposite sides to 'count' where the point G should be).*

Solutions:

a) $DE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$DE = \sqrt{(-3-3)^2 + (3+5)^2}$

$DE = \sqrt{36+64}$

$DE = \sqrt{100} = 10$

b) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_{DE} = \dfrac{3+5}{-3-3}$

$m_{DE} = -\dfrac{8}{6} = -\dfrac{4}{3}$

c) $m_{DE} = -\dfrac{4}{3}$

$\therefore m_{EF} = \dfrac{3}{4}$ ($\perp$ lines)

$\dfrac{3}{4} = \dfrac{-5-k}{3+1}$

$12 = 4(-5 - k)$

$12 = -20 - 4k$

$4k = -32$

$k = -8$

d) $M\left(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}\right)$

$M\left(\dfrac{-3-1}{2}; \dfrac{3-8}{2}\right)$

$M\left(-2; -\dfrac{5}{2}\right)$

e) $E(3;-5)$ $\quad$ $M\left(-2; -\dfrac{5}{2}\right)$ $\quad$ $G(x;y)$

$\dfrac{3+x}{2} = -2$ $\quad$ $\dfrac{-5+y}{2} = -\dfrac{5}{2}$

$3 + x = -4$ $\quad$ $-5 + y = -5$

$x = -7$ $\quad$ $y = 0$ $\quad$ $\therefore G(-7;0)$

Example 2

a) C is the point (1 ; –2). The point D lies in the second quadrant and has coordinates (x ; 5). If the length of CD is $\sqrt{53}$ units, find the value of x.

b) Given the points P (5 ; –1) and Q (2 ; a), find a if the gradient of PQ is 2.

Teaching notes:

Remind learners that they should always draw a sketch if one is not given. Draw a sketch with learners now. Even though the point (x ; 5) cannot be plotted precisely, draw a dashed line representing $y = 5$ to show that the point given will lie on that line as it has a y-coordinate of 5.

a)

Ask: *What aspect of Analytical Geometry is required to answer this question?*

(Distance).

Ask: *What makes this question different from other distance questions?*

(It needs to be used in reverse – the distance is given and a coordinate needs to be found).

Note: When you get to the part in the solution which shows that the equation that needs to be solved is a quadratic equation, discuss this with learners.

Ask: *Why will there be two solutions when it seems clear that there should only be one solution in the question?*

(There are two positions possible for a point that would make CD = $\sqrt{53}$. Discuss this with learners and show this on the sketch).

Once the equation has been solved, show again why two answers are possible but remind learners that the questions stated the point was in quadrant 2 and therefore one solution will not be possible.

b)

Tell learners to draw a sketch. When they have drawn their own sketch, draw a sketch on the board. Even though the point (2 ; a) cannot be plotted precisely, draw a dashed line representing $x = 2$ to show that the point given will lie on that line as it has a x-coordinate of 2.

Ask: *What aspect of Analytical Geometry is required to answer this question?*

(Gradient).

Ask: *What makes this question different from other gradient questions?*

(It needs to be used in reverse – the gradient is given and a coordinate needs to be found).

Solutions:

a) CD = $\sqrt{53}$

$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{53} = \sqrt{(x-1)^2 + (5+2)^2}$

$53 = (x - 1)^2 + (5 + 2)^2$

$53 = x^2 - 2x + 1 + 49$

$0 = x^2 - 2x - 3$

$0 = (x + 1)(x - 3)$

$\therefore x = -1$ or $x = 3$

$\therefore x = -1$ (2nd quadrant)

b) $m_{PQ} = 2$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$2 = \dfrac{a + 1}{2 - 5}$

$-6 = a + 1$

$\therefore a = -7$

8. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

9. Give learners an exercise to complete with a partner.

10. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=0C_OJjK1VQw

https://www.mathsisfun.com/algebra/distance-2-points.html

http://www.mathwarehouse.com/algebra/distance_formula/index.php
(Distance)

http://www.teacherschoice.com.au/Maths_Library/Gradient/gradient_-_two_fixed_points.htm

http://www.coolmath.com/algebra/08-lines/06-finding-slope-line-given-two-points-01

https://www.youtube.com/watch?v=QW2yT-AtsA0
(Gradient)

https://www.youtube.com/watch?v=ZXoJSzmaZ4E

http://virtualnerd.com/algebra-1/radical-expressions-equations/distance-midpoint-formulas/mid-point-formula/midpoint-between-coordinates

(Midpoint)

https://www.youtube.com/watch?v=434AtgfoeKc

(Application of the distance formula)

TERM 1, TOPIC 4, LESSON 2

FINDING EQUATIONS OF STRAIGHT LINES

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	31
Lesson Objectives By the end of the lesson, learners should be able to find the equation of a straight line given: • two points • one point and information regarding the gradient (parallel or perpendicular).	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 9 from the Resource Pack ready for use during lesson.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson draw a Cartesian plane with three coordinates, A, B and C plotted from point 2.
6. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5 6	72 74	1	61	5-9	66-71	4.5	99	4.2 4.3 4.4	115 119 123

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have encountered this topic before. They found equations of straight lines in Grades 9 and 10. This should give them confidence going forward.
2. Some of the prescribed textbooks have presented angle of inclination before finding the equations of lines. If your school uses one of these textbooks, look at the exercises carefully before giving them to learners to do. If inclination has already been done, it could also come up in the exercise on finding equations of straight lines. Tell learners to leave these until after you have taught it.

DIRECT INSTRUCTION

1. Say: Finding the equation of a straight line is a concept covered in functions as well as Analytical Geometry. In the past, we have used the form $y = mx + c$ or $y = ax + q$ as our base. Today, we are going to look at a second option available to us to find the equation of a straight line.
2. Ask learners to write down the formula to find gradient.

$$\left(m = \frac{y_2 - y_1}{x_2 - x_1}\right)$$

Say: Change the format of the equation so that there are no fractions.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

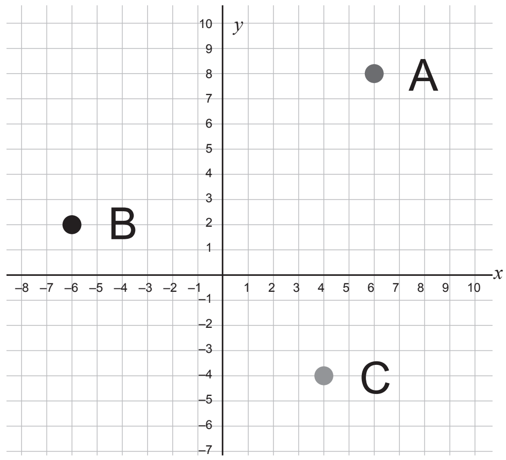
$$m(x_2 - x_1) = y_2 - y_1$$

3. Tell learners that this equation: $y_2 - y_1 = m(x_2 - x_1)$, can be used to find the equation of a straight line. Because one of these points can represent any point, there is no need to label it as above.

We can therefore state the equation as: $y - y_1 = m(x - x_1)$

4. By substituting the gradient and one point, the equation of any straight line can be found.
5. Say: *This formula is given on the Grade 12 formula sheet.*
6. Ask learners to copy the following into their books: (the diagram is available in the Resource Pack – Resource 9).

$A(6 ; 8)$; $B(-6 ; 2)$; $C(4 ; -4)$



7. Say: We are going to use this method to find the equation of line BC.

<table>
<tr>
<td>
Solution:
$B(-6 ; 2)$ $C(4 ; -4)$
$m_{BC} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$m_{BC} = \dfrac{-4 - 2}{4 - (-6)}$
$m_{BC} = \dfrac{-6}{10} = \dfrac{3}{5}$
$y - y_1 = m(x - x_1)$
$y - 2 = -\dfrac{3}{5}(x - (-6))$
$y = -\dfrac{3}{5}(x + 6) + 2$
$y = -\dfrac{3}{5}x - \dfrac{18}{5} + 2$
$y = -\dfrac{3}{5}x - \dfrac{8}{5}$
</td>
<td>
Teaching notes:
Say: To find the equation of any straight line, we need the gradient and a point.
If two points are given, then the gradient needs to be found first.
Tell learners that they can use ANY point to find the equation of the line.
Once you have completed the example with learners, ask them to re-do the question but to use C when finding the equation to confirm that they get the same answer.
</td>
</tr>
<tr>
<td colspan="2">
Using point C:
$m = \dfrac{3}{5}$ C(4 ; –4)
$y - y_1 = m(x - x_1)$
$y - (-4) = -\dfrac{3}{5}(x - 4)$
$y + 4 = -\dfrac{3}{5}x + \dfrac{12}{5}$
$y = -\dfrac{3}{5}x + \dfrac{12}{5} - 4$
$y = -\dfrac{3}{5}x - \dfrac{8}{5}$
</td>
</tr>
</table>

8. Ask learners to find the equation of line AB. Tell them to do it twice again using point A first and then point B.

Solutions:

$$A(6\ ;\ 8) \qquad B(-6\ ;\ 2)$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{2-8}{-6-6}$$

$$m_{AB} = \frac{-6}{-12}$$

$$m_{AB} = \frac{1}{2}$$

$m_{AB} = \frac{1}{2} \quad A(6;8)$ $y - y_1 = \mathrm{m}(x - x_1)$ $y - 8 = \frac{1}{2}(x - 6)$ $y = \frac{1}{2}x - 3 + 8$ $y = \frac{1}{2}x + 5$	$m_{AB} = \frac{1}{2} \quad B(-6\ ;\ 2)$ $y - y_1 = \mathrm{m}(x - x_1)$ $y - 2 = \frac{1}{2}(x - (-6))$ $y = \frac{1}{2}(x + 6) + 2$ $y = \frac{1}{2}x + 3 + 2$ $y = \frac{1}{2}x + 5$

9. Discuss the following with learners:

Always check if the answer looks correct.

- If the line is sloping up then the gradient should be positive
- If the line is sloping down then the gradient should be negative
- Check that the y-intercept looks correct.

10. Once you feel confident that learners know how to find the equation of a straight line when the gradient and a point are known, move on to other types of questions involving straight lines. Tell learners to take notes as you discuss other aspects of straight lines.

11. Ask: *What is the equation of the x-axis?*
    ($y = 0$)
    What is the equation of the y-axis?
    ($x = 0$)

12. Say: *All horizontal lines are in the form $y = c$ and all vertical lines are in the form $x = c$, where c is always a constant.*

13. Ask: *What is the gradient of any horizontal line?*
    (Zero).
    What is the gradient of any vertical line?
    (Undefined).

14. Ask: *What can you tell me about lines that have the same gradient?*
(The lines are parallel).
Ask: *What can you tell me about the relationship between perpendicular lines and their gradients?*
(The product of their gradients is –1).
Do the following examples with learners.
Learners should write the examples in their books, making notes as they do so.

Examples:	Teaching notes:
Find the equation of a line that goes through the point (–1;3) and is: a) Parallel to the x-axis b) Perpendicular to the x-axis Solutions: a) $y = 3$ b) $x = –1$ Note: Tell learners to draw a quick sketch of these two lines to confirm the information given in the question.	a) Ask: *What kind of line is parallel to the x-axis?* (A horizontal line). Ask: *What is the standard form of a horizontal line?* (y = c) Ask: *Which part of the coordinate is of importance to us?* (The y coordinate). b) Ask: *What kind of line is perpendicular to the x-axis?* (A vertical line). Ask: *What is the standard form of a vertical line?* (x = c). Ask: *Which part of the coordinate is of importance to us?* (The x coordinate).

Find the equation of a line passing through the point (1;–5) that is a) parallel to the line $2y = x + 4$ b) perpendicular to the line $2y = x + 4$ Solutions: a) $2y = x + 4$ $\frac{2y}{2} = \frac{x}{2} + \frac{4}{2}$ $y = \frac{1}{2}x + 2$ $m = \frac{1}{2}$ (1;-5) $y - y_1 = m(x - x_1)$ $y - (-5) = \frac{1}{2}(x - 1)$ $y + 5 = \frac{1}{2}x - \frac{1}{2}$ $y = \frac{1}{2}x - \frac{11}{2}$ b) $m = \frac{1}{2}$ $\therefore \perp$ $m = 2$ (1;-5) $y - y_1 = m(x - x_1)$ $y - (-5) = -2(x - 1)$ $y + 5 = -2x + 2$ $y = -2x - 3$	Say: *When asked to find the equation of a straight line (that is not vertical or horizontal), always remind yourself that you need the gradient and a point.* *In the case of this question, you have been given a point. You therefore need to find the gradient.* a) Ask: *How can you find the gradient from the information given?* (Get the equation given into standard form so that you can see the gradient). b) Ask: *How can you find the gradient from the information given?* (Get the given equation into standard form so that you can see the gradient, then find the reciprocal of the gradient and change the sign because the products of the gradients must equal –1). Once the solutions have been found, ask learners to sketch the original line given on a Cartesian plane as well as the 2 lines found to check that they look correct according to the information given in the question.

15. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

16. Give learners an exercise to complete on their own.

17. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=9hryH94KFJA

https://www.youtube.com/watch?v=pJ0_Lvkvo9E

https://www.youtube.com/watch?v=7G8EwEc5xLw

TERM 1, TOPIC 4, LESSON 3

INVESTIGATION

Suggested lesson duration: 1 hour

POLICY AND OUTCOMES

A

CAPS Page Number	31
Lesson Objectives By the end of the lesson, learners will have: ● completed an investigation on the angle of inclination.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the investigation. The investigation is provided in the Resource Pack – Resource 12.
3. Make a copy of the investigation for each learner.

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. An investigation is an activity that should lead a learner to a deeper understanding of a mathematical concept.
2. This investigation deals with the angle of inclination.

DIRECT INSTRUCTION

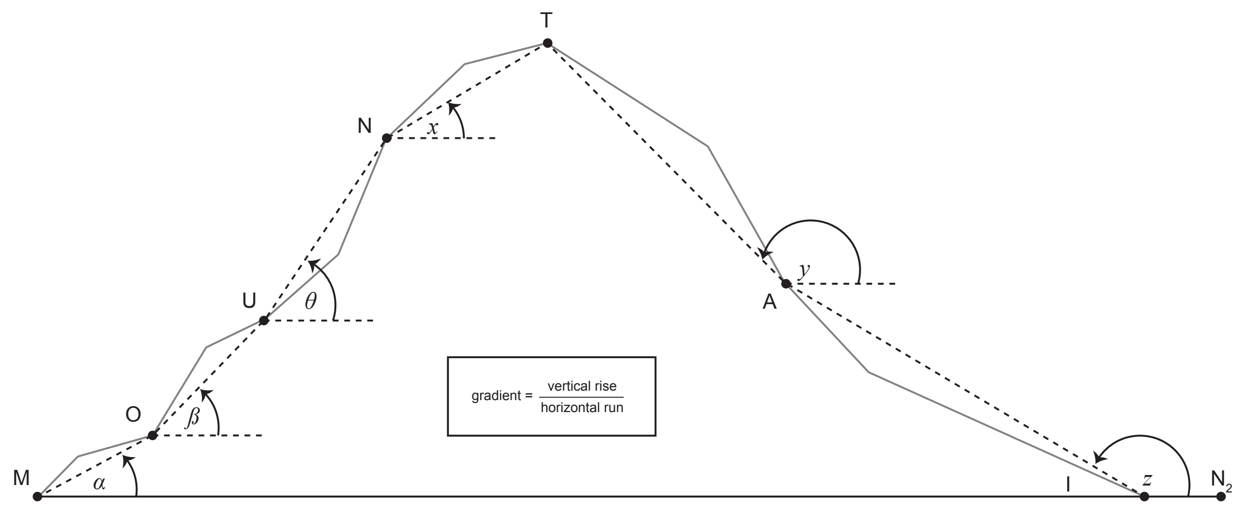
1. Hand out the investigation to each learner.
2. Tell learners that they must work on their own and that they will have 1 hour to complete the investigation.

3. Tell learners the mark they receive will count 15% towards their school-based assessment mark for the end of the year.

C

MARKING MEMORANDUM FOR INVESTIGATION

Investigation – Analytical Geometry



Look carefully at the drawing of the mountain represented above. Imagine walking up the mountain from M and down the other side to N_2.

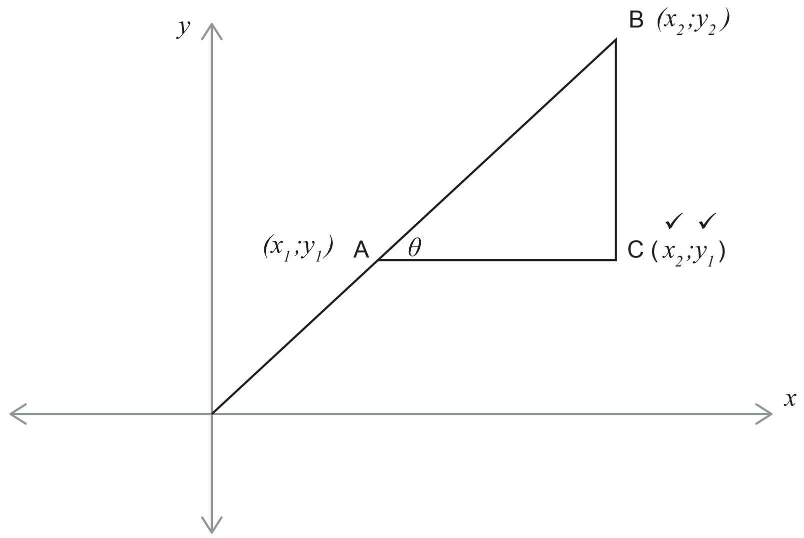
The dashed lines represent an average gradient between two points. Note that on the way up the mountain (M,O,U,N,T) all the gradients are positive and on the way down the mountain (A,I) all the gradients are negative. The flat path at the end, to N_2, has a gradient of zero.

Note the angles made ($\alpha, \beta, \theta, x, y, z$). They are formed from a horizontal line (as angles are drawn if you use a protractor), in a clockwise direction to the line made representing the gradient. These are called angles of inclination.

Underline the correct answers in the following statements and fill in the missing spaces:

1.	The lines MO, OU, UN and NT all slope **up/down** ✓ and therefore have a **positive/negative** ✓ gradient. The angles of inclination made by the lines MO,OU,UN and NT all lie between **0º** and **90º** ✓ and are therefore **acute** ✓ angles.	(4)
2.	The lines TA and AI all slope **up/down** ✓ and therefore have a **positive/negative** ✓ gradient. The angles of inclination made by the lines TA and AI all lie between **90º** and **180º** ✓ and are therefore **obtuse** ✓ angles.	(4)

Study the diagram below, then answer the questions that follow.
Use your knowledge of Grade 10 Analytical Geometry and Trigonometry,



3. Fill in the coordinate at C in terms of x and y. (Points A and B will be useful) (2)

4. In ΔABC,
BC (distance) = $y_2 - y_1$ ✓ AC (distance) = $x_2 - x_1$ ✓ ✓ (3)

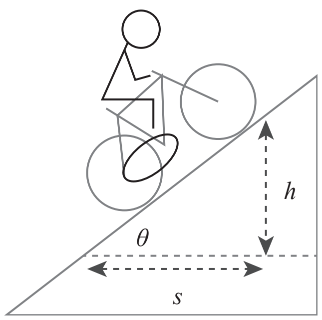
5. From trigonometry, you know that:
$\tan\theta = \dfrac{y}{x}$ or $\dfrac{opp}{adj}$ ✓

(use names of sides or $x/y/r$) (1)

∴ (using your answers from ABOVE (4), $\tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ ✓ ✓ (2)

This is how you find the angle of inclination.

Using the gradient formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, which is essentially made up of $\dfrac{adj}{hyp}$ assists you in finding the angle of inclination of a straight line.

	$\tan\theta$ = gradient $\left(\dfrac{h}{s}\right)$ $\tan\theta = m$ (where θ is the angle of inclination and m is the gradient of the line)

Use the diagram of the mountain and the formula from above, answer the following questions:
(the drawing is not drawn to scale – the angles cannot be measured)

6. If the first part of the path (MO) rises 3m over a horizontal distance of 27m, find the angle of inclination(α). (3)

$$\tan\alpha = \frac{3}{27} \checkmark\checkmark$$

$$\therefore\ \alpha = 6{,}34° \checkmark$$

7. Find the angle of inclination (θ) of UN if it rises 35m over a horizontal distance of 2m. (3)

$$\tan\theta = \frac{35}{2} \checkmark\checkmark$$

$$\therefore\ \theta = 86{,}73° \checkmark$$

You will learn how to find an angle of inclination which is linked to a line with a negative gradient in class in a later lesson.

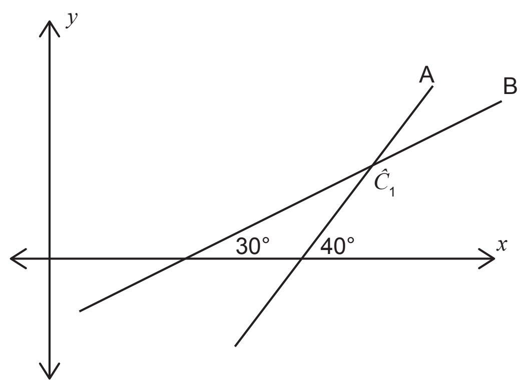
8. Underline the correct answers in each statement:

   a. If the gradient of a line is 0 (zero), the line will be **<u>horizontal</u>/vertical**. ✓
   The inclination of this line is **<u>0º</u>**, **90º**, **180º** ✓ (2)

   b. If the gradient of a line is undefined (perpendicular), the line will be **horizontal/<u>vertical</u>** ✓
   The inclination of this line is **0º**, **<u>90º</u>**, **180º** ✓ (2)

9. Use the diagram below to answer the questions that follow:

   Line A has an inclination of 40°.
   Line B has an inclination of 30°.



   a) Find the gradient of line A and line B. Round your answer to TWO decimal places. (2)

   $m_A = 0{,}84$ ✓ $m_B = 0{,}58$ ✓

   b). i. Find $\hat{C}_1$

   $$\hat{C}_1 = 170° \checkmark$$

   ii. What theorem(s) did you use? Exterior angle of a triangle is equal to the sum of the opposite interior angles and angles on a straight line add up to 180º ✓ (1)

Total: 30 marks

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=0C_OJjK1VQw

https://www.mathsisfun.com/algebra/distance-2-points.html

http://www.mathwarehouse.com/algebra/distance_formula/index.php

TERM 1, TOPIC 4, LESSON 4

ANGLE OF INCLINATION

Suggested lesson duration: 3 hours

A POLICY AND OUTCOMES

CAPS Page Number	31
Lesson Objectives By the end of the lesson, learners should be able to: • find the inclination of a line.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 10 from the Resource Pack ready for use during lesson.
4. Write the lesson heading on the board before learners arrive.
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	67	2	66	1	63	4.2	88	4.5	126
3	69			2	64	4.3	93	4.6	130
4	70			3	64	4.4	94	4.7	136
				4	65			4.8	141

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

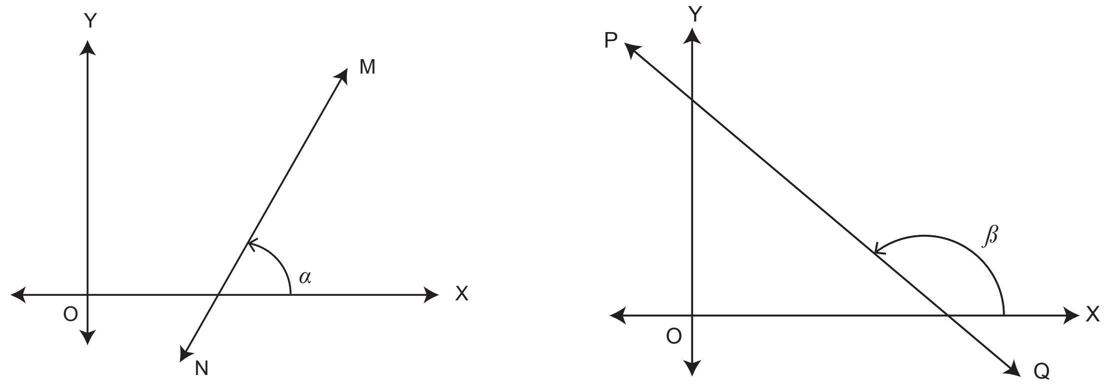
1. This is the only new concept in this topic for Grade 11.
2. Learners completed an investigation in the previous lesson as a form of introduction to the concept of angle of inclination.
3. The investigation has the added advantage of revising some trigonometry skills from Grade 10 and will be beneficial for learners when they do trigonometry next term.

DIRECT INSTRUCTION

1. Say: *Although you have just completed an investigation on the angle of inclination and probably have a good sense of what it is and how to find it, I am going to work from the beginning to ensure that you have a good understanding of how to find the inclination of a line.*
2. Draw the following diagrams on the board and discuss them with learners.
   Give learners the definition of the angle of inclination and ask them to write it in their books.
   The sketches are available in the Resource Pack – Resource 10.

<u>The angle of inclination</u>
The angle between a line and the horizontal line (most often the x-axis).
Can be any measurement from 0º to 180º.
Always measured from the horizontal line in an anti-clockwise direction.
If the line has a positive gradient, the angle of inclination will be less than 90º.
If the line has a negative gradient, the angle of inclination will be between 90º and 180º.

   On the first diagram show how the angle starts on the horizontal (the x-axis in this case) and moves in an anti-clockwise direction until it meets the line in question.
   Show the same on the second diagram.
   Point out that when the line has a positive slope, the angle of inclination will be acute and when the line has a negative slope the angle of inclination will be obtuse.



3. Tell learners to make a sketch of the two diagrams in their books.

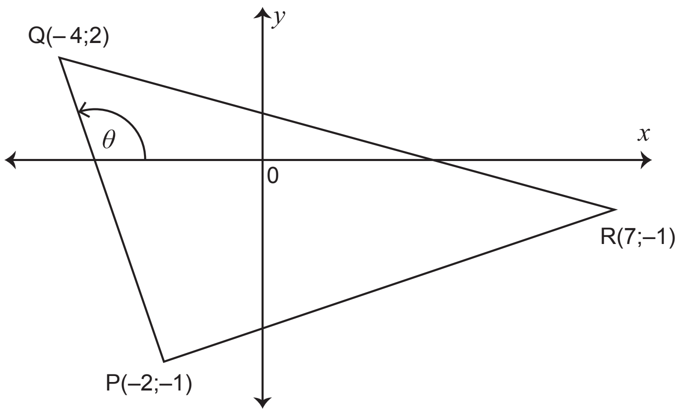
4. Ask: *What is the formula for finding the angle of inclination?*
   (tan $\theta = m$).

5. Ask: *Why is the trigonometric ratio tangent always used and not sine or cosine?*
   (The angle of inclination is directly linked to the gradient, which is 'change in y' divided by the 'change in x' – this is $\frac{y}{x}$ or $\frac{opp}{hyp}$ which is the tangent ratio).

6. Say: *I am going to do some fully worked examples with you now. Write them in your book and make your own notes where necessary.*

Example 1

Find the angles of inclination of all three line segments (QP, QR and PR)



Teaching notes:

Ask: *What procedure do we need to follow to find the angle of inclination?*

(Find the gradient).

Ask: *Are you expecting the gradient of PQ to be positive or negative?*

(Negative – it slopes down).

Ask: *What does this tell you about the angle of inclination?*

(It will be an obtuse angle).

Ask: *Are you expecting the gradient of QR to be positive or negative?*

(Negative – it slopes down).

Ask: *What does this tell you about the angle of inclination?*

(It will be an obtuse angle).

Ask: *Can we find the angle of inclination of a line that doesn't cross the x-axis?*

(Yes – the angle of inclination is from any horizontal up to the line).

Point out that we could draw in a horizontal line to cross the line PR or we could lengthen it to show that it can cross the x-axis. No matter which way we look at it, the angle of inclination will be the same because the gradient of the line is constant, and the angle of inclination is directly linked to gradient.

Ask: *Are you expecting the gradient of PR to be positive or negative?*

(Positive – it slopes up).

Ask: *What does this tell you about the angle of inclination?*

(It will be an acute angle).

Solution:

Angle of inclination of PQ:

$\tan \theta = m$

$\tan \theta = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\tan \theta = \dfrac{2 - (-1)}{-4 - (-2)}$

$\tan \theta = \dfrac{3}{-2}$

RA = 56,3º

$\therefore\ \theta = 180º – 56,3º = 123,7º$

Angle of inclination of QR:

Let the angle of inclination be β

$\tan \beta = m$

$\tan \beta = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\tan \beta = \dfrac{2 - (-1)}{-4 - 7}$

$\tan \beta = \dfrac{3}{-11}$

RA = 15,3º

$\therefore\ \beta = 180º – 15,3º = 164,7º$

Angle of inclination of PR:

Let the angle of inclination be α

$\tan \alpha = m$

$\tan \alpha = \frac{y_2 - y_1}{x_2 - x_1}$

$\tan \alpha = \frac{-1-(-1)}{7-(-2)}$

$\tan \alpha = 0$

$\therefore \ \alpha = 0°$

Example 2

Three points, A(-1;2),B(-3;-2) and C(1;-1) are given.

Plot the points on a Cartesian plane and find the angle $B\hat{A}C$ correct to 2 decimal places.

Teaching notes:

Tell learners to first draw their Cartesian plane and plot the three points given.

Say: *Draw in AB and AC to form the angle that needs to be found.*

Ask: *Is $B\hat{A}C$ an angle of inclination?*

(No).

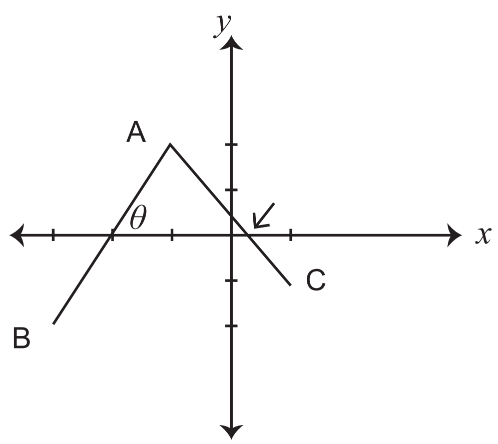
Ask: *Then how will we find the size of this angle?*

(By finding the angles of inclination of the two lines drawn in – AB and AC, then using the exterior angle of a triangle theorem).

Point out that if learners are ever asked to find the size of an angle that is not an angle of inclination it will usually require them to find one or more angles of inclination, then use grade 8 geometry to assist them.

Solution:

Tell learners to label the two angles of inclination θ and α as shown below.



$\tan \theta = m$

$\tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$

$\tan \theta = \frac{-2 - 2}{-3 - (-1)}$

$\tan \theta = \frac{-4}{-2}$

$\tan \theta = 2$

$\therefore \ \theta = 63{,}43^\circ$

$\tan \alpha = m$

$\tan \alpha = \frac{y_2 - y_1}{x_2 - x_1}$

$\tan \alpha = \frac{-1 - 2}{1 - (-1)}$

$\tan \alpha = \frac{-3}{2}$

RA $= 56{,}31^\circ$

$\therefore \ \alpha = 180^\circ - 56{,}31^\circ = 123{,}69^\circ$

$B\hat{A}C = 123{,}69^\circ - 63{,}43^\circ$ ext < of Δ

$\therefore \ B\hat{A}C = 60{,}26^\circ$

Example 3

A line has an angle of inclination of 110°. State the gradient of the line, correct to 2 decimal places.

Teaching notes:

Ask: *How can you find the gradient of a line if you know the angle of inclination?*

(Use the formula, $\tan \theta = m$ and substitute the angle).

Solution:

$\tan \theta = m$

$\tan 110^\circ = m$

$\therefore \ m = -2{,}75$

7. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
8. Give learners an exercise to complete on their own.
9. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=IHLG-WXRECw

http://learn.mindset.co.za/sites/default/files/resourcelib/emshare-topic-overview-asset/maths-11-1-guide-analytical-geometry-cartesian-plane_0.pdf

TERM 1, TOPIC 4, LESSON 5

REVISION AND CONSOLIDATION

Suggested lesson duration: 3 hours

A POLICY AND OUTCOMES

CAPS Page Number	31
Lesson Objectives By the end of the lesson, learners should be able to: • apply and combine the knowledge and skills learned in the previous lessons.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 11 from the Resource Pack ready for use in the lesson.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson have the tips from point 3 ready.
6. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
7 8 Rev S Ch	76 79 80 82	Rev	67	Qu's Ch 4	72 75	46 Rev	103 106	4.9	143

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Use any time left over well – learners must spend as much time as possible revising and consolidating their knowledge.
2. One of the points made in the diagnostic reports was that learners struggled to answer questions if topics were combined within one question. Continue to point out to learners that this is common.

DIRECT INSTRUCTION

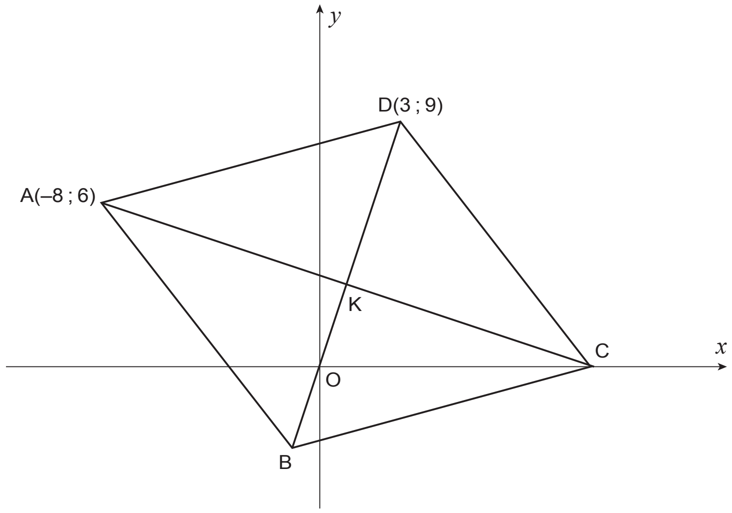
1. Ask learners to recap what they have learned in this section. Point out issues that you know are important as well as problems that you encountered from your own learners during the topic.
2. If learners want you to explain a concept again, do that now.
3. Spend some time discussing the following with learners:

> Tell learners the following concerning Analytical geometry questions in assessments:
> 1. You are always given the formulae so there is no need to lose marks because you couldn't remember it.
> 2. If a question is set out differently and you are unsure where to start, ask yourself:
>   - Would knowing the length of a line help me answer the question? (<u>Distance</u>).
>   - Would knowing how steep the line is help me answer the question? (<u>Gradient</u>).
>   - Would knowing where the middle of the line is help me answer the question? (<u>Mid-point</u>).
>   - Would knowing the <u>angle of inclination</u> help me answer the question?
>   - Would finding the <u>equation of a line</u> help me answer the question?
> 3. If there isn't a sketch, ALWAYS draw one!

4. Do two examples from past exam papers in full with learners.
   Learners should write the examples in their books, making notes as they do so.
   All diagrams in the examples below are available in the Resource Pack – Resource 11.

Example 1

In the diagram $A(-8;6)$, B, C and $D(3;9)$ are the vertices of a rhombus. The equation of BD is $3x - y = 0$. The diagonals of the rhombus intersect at point K.



a) Calculate the perimeter of ABCD. Leave your answer in simplest surd form.
b) Determine the equation of diagonal AC in the form $y = mx + c$.
c) Calculate the coordinates of K if the equation of AC is $x + 3y = 10$
d) Calculate the coordinates of B.
e) Determine, showing ALL your calculations, whether rhombus ABCD is a square or not.

NOV 2014

Teaching notes:

Point out that to answer this question well, it is essential to know the properties of quadrilaterals (in this case a rhombus). It will not be possible to answer a), b), d) and e) without this knowledge.

a)

Ask: *What aspect of Analytical geometry will we need to use to find perimeter?*

(Distance).

Ask: *Will we need to find the length(distance) of each side?*

(No, a rhombus has 4 equal sides).

b)

Ask: *What do we need to find the equation of a straight line?*

(Gradient and a point).

Ask: *Do we have any of these?*

(A point is given).

Ask: *How will we find the gradient?*

(The equation (and therefore the gradient) of diagonal BD is given. The diagonals of a rhombus bisect at right angles which means we can find the gradient of the other diagonal AC).

c)

Ask: *What happens at K?*

(The diagonals meet – it is the point of intersection of the diagonals).

Say: One way will be to use simultaneous equations.

Ask: *What is a second way of finding the point of intersection?*

(Find the x-intercept of AC then find the midpoint of A and C).

d)

Ask: *How can we find the coordinate of B?*

(Use finding the midpoint in reverse because K is the midpoint of BD).

e)

Ask: *What would be required to make a rhombus into a square?*

(The angles would have to be right angles).

Ask: *How can we use Analytical Geometry to check if this is the case?*

(Find the gradient of two meeting sides and find the product of the gradients. If the product is –1 then they are perpendicular).

Solutions

a) $AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AD = \sqrt{(9-6)^2 + (3+8)^2}$

$AD = \sqrt{(9+121)}$

$AD = \sqrt{130}$

$\therefore$ the perimeter of rhombus ABCD is $4\sqrt{130}$

b) BD: $y = 3x$

$\therefore$ m = 3

$\therefore \perp$ m $= -\frac{1}{3}$ A(–8;6)

$y - y_1 = m(x - x_1)$

$y - 6 = -\frac{1}{3}(x - (-8))$

$y = -\frac{1}{3}(x + 8) + 6$

$y = -\frac{1}{3}x - \frac{8}{3} + 6$

$y = -\frac{1}{3}x + \frac{10}{3}$

c) $y = 3x$ and $y = -\frac{1}{3}x + \frac{10}{3}$

$3x = -\frac{1}{3}x + \frac{10}{3}$

$9x = -x + 10$

$10x = 10$

$x = 1$

$y = 3(1) = 3$

$\therefore$ K(1;3)

d) D(3;9) K(1;3) B(x;y)

$\frac{3+x}{2} = 1$ $\frac{9+y}{2} = 3$

$3 + x = 2$ $9 + y = 6$

$x = -1$ $y = -3$

$\therefore B(-1;-3)$

e)

$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 - 6}{3 + 8}$ $= \frac{6 + 3}{-8 + 1}$

$= \frac{3}{11}$ $= -\frac{9}{7}$

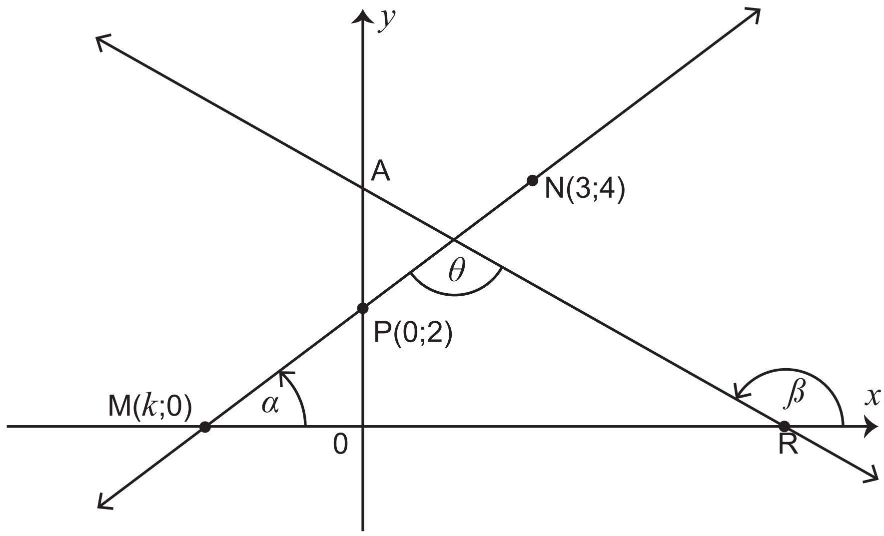
$\frac{3}{11} \times -\frac{9}{7} \neq -1$ $\therefore B\hat{A}D \neq 90^\circ$

$\therefore$ ABCD is not a square

Example 2

In the diagram, R and A are the x- and y-intercepts respectively of the straight line AR. The equation of AR is $y = -\frac{1}{2}x + 4$. Another straight line cuts the y-axis at $P(0;2)$ and passes throught he pints $M(k\ ;\ 0)$ and $N(3\ ;\ 4)$.

α and β are angles of inclination of the lines MN and AR respectively.



a) Given that M, P and N are collinear points, calculate the value of k.
b) Determine the size of θ, the obtuse angle between the two lines.
c) Calculate the length of MR.
d) Calculate the area of ΔMNR.

NOV 2016

Teaching notes:

a)

Ask: *What does collinear mean?*

(In a straight line. The gradients of each pair of coordinates are equal).

b)

Say: *Remember that when asked to find an angle that is NOT an angle of inclination, you need to use some Grade 8 geometry knowledge as well as find one or more angles of inclination.*

Ask: *What do you need to find in this case, and what Grade 8 geometry theorem will you use?*

(Find the two angles of inclination α and β, then use exterior angle of a triangle to find θ).

c)

Ask: *What do you need to know before you can find the length of MR?*

(The coordinate of R).

Ask: *How can you find the coordinate of R?*

(By finding the the x-intercept of line AR).

Point out that although we can then use the distance formula, the length could also be found by counting as both points are on the x-axis.

d)

Ask: *How do you find the area of a triangle?*

(Use the formula ($\frac{1}{2}$ base × $\perp$ height)).

Ask: *Do you know the base and the height?*

(We know the base).

Ask: *How can you find the height?*

(The point N is known – the height is given by the y-value).

Solutions:

a) $m_{MP} = m_{PN}$

$\frac{2-0}{0-k} = \frac{4-2}{3-0}$

$\frac{2}{-k} = \frac{2}{3}$

$-2k = 6$

$k = -3$

b) $\tan\alpha = m_{PN}$

$\tan\alpha = m_{PN}$ $\tan\alpha = \frac{2}{3}$ $\therefore\ \alpha = 33{,}69^\circ$	$\tan\beta = m_{AB}$ $\tan\beta = -\frac{1}{2}$ $\therefore\ \beta = -26{,}57^\circ + 180^\circ$ $\therefore\ \beta = 153{,}43^\circ$

$\therefore\ \theta = 153{,}43^\circ - 33{,}69^\circ$ (ext < of Δ)

$\therefore\ \theta = 119{,}74^\circ$

c) $y = -\frac{1}{2}x + 4$

x-int; $y = 0$

$0 = -\frac{1}{2}x + 4$

$0 = -x + 8$

$x = 8$ $\therefore$ R(8;0)

$\therefore$ MR = 11 units

d) $b = 11$ $ht = 4$

Area ΔMNR $= \frac{1}{2}$ base $\times \perp$height

$= \frac{1}{2}(11)(4)$

$= 22units^2$

4. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

5. Give learners an exercise to complete with a partner.

6. Walk around the classroom as learners do the exercise. Support learners where necessary.